Mathematical Neighbours

Introduction: Today we are going to solve some puzzles, draw some pictures, and colour some maps. While doing these activities, we will be learning a new and colourful way of representing real-life situations.

Task 1A: One king and his kingdom!

Once upon a time, there was a kingdom ruled by a king who had two daughters and one son. It was his wish that upon his death, this kingdom should be divided into three connected regions, one region for each child, such that all his three children are each other's neighbours. And the regions need not be equal.

Note: Being neighbours means sharing some common border, which is more than just a single point. Also, the shares of each of the king's children need not be equal or of the same shape.

In this map, region A and region B are not neighbours as they share only one point as a common border. Hence the king's wish
is not satisfied.

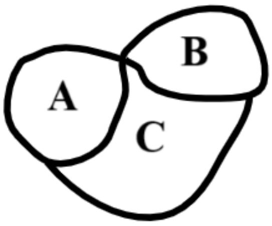
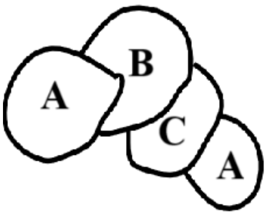


Figure 1



In this map, region A is a neighbour of region B and region C but region A is divided in two parts that are not connected to each other. Hence the king's wish is not satisfied.

Figure 2

Q. Can you draw another map such that the king's wish is satisfied? Compare solutions designed by your classmates.

Task 1B: Second king and his kingdom!

Now, there was another kingdom ruled by another king who had two daughters and two sons. He also had a similar last wish. It was his wish that upon his death, this kingdom should be divided into four connected regions, one region for each child, such that all his four children are each other's neighbours.

Note: Being neighbours means sharing some common border, which is more than just a single point. Also, the shares of each of the king's children need not be equal or of the same shape.

Q. Do you think that this is possible? Can you draw a map below such that the king's wish is satisfied? Draw a kingdom of any shape and look for a possible solution satisfying the King's wish. Is your solution similar to your classmates?

Task 1C: Yet another king and his kingdom!

There was a third kingdom ruled by another king who had three daughters and two sons. His wish was also similar to the earlier two kings. Like the other two kings, he also wished that upon his death, this kingdom should be divided into five connected regions, one region for each child, such that everyone is everyone's neighbour.

Note: Being neighbours means sharing some common border, which is more than just a single point. Also, the shares of each of the king's children need not be equal.

Q. Do you think that this is possible? Draw a kingdom of any shape and look for a possible solution satisfying the King's wish. Compare solutions designed by your classmates.

Q. Is there any difference between Tasks 1A, 1B and 1C, apart from the number of children?. Could you draw maps for all the three? If not, why was there a difference in the result? Discuss with your group members.

Let us now try to verify our conclusion through a new representation of the same problem.

Task 2: Representing maps

A 'node' (drawn as a dot or point) represents an individual element (i.e., member) of the set of things in front of us. A 'link' (drawn as a connecting line) represents that the two nodes joined by that link are connected to each other. If there is no link connecting a pair of nodes, that means those two nodes are not linked to each other.

Let us call this new representation containing nodes and links a 'Network Diagram'.

Look at the regions and their Network Diagram given below. Let us say a kingdom is divided in three parts (A, B, and C) as shown in the diagram on the left. In the diagram on the right, each part of the kingdom is represented by a node (point) and the node pairs are connected by a link whenever the two regions share a common border.

In the diagram on the left, region A shares a border with regions B and C. Thus, in the diagram on the right, point A is linked to both B and C. But regions B and C do not share a border. Hence, in the Network Diagram, we can see that there is no link between nodes B and C.

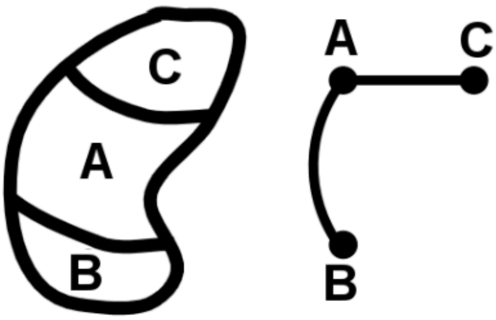


Figure 3

Note that the links need not be straight lines, they can be curved as well like the one in the network diagram drawn above.

In Task 1, when you draw the Network Diagrams for the maps drawn by you, the regions of the kingdom will become the nodes and if two regions share a common border then those two nodes will have a link between them.

Note that the link (connector) may not always be a straight line.

Q. Draw Network Diagrams for all the solutions you got for Task 1A. What can you say about the Network Diagrams of different solutions to the same problem? Are all the Network Diagrams of Task 1A the same?

Q. Draw Network Diagrams for all the solutions you got for Task 1B. What is your observation about the Network Diagrams of different solutions to the same problem?

In Task 1C, we were unable to draw a map. But if such a map existed then how would its Network Diagram look? Like all the earlier network diagrams, all nodes will have to be connected by a link here too. If there is a solution, then the network diagram for Task 1C will look like this.

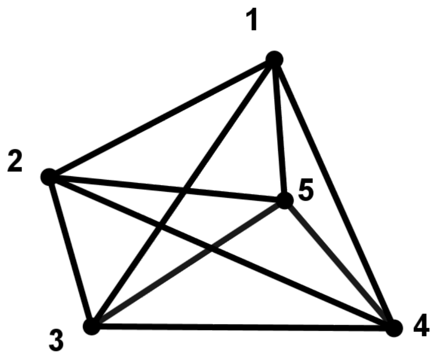


Figure 4

You can notice that in this network diagram the links are crossing each other. Can you draw this Network Diagram for Task 1C without crossing the links (no links crossing each other)?

Task 3: Colouring Maps

Task 3A: Another country!

See the map of an imaginary country below, where boundaries between different states are marked by dotted lines. Can you colour it in such a way that every state can be clearly distinguished from its neighbours?

Q. Before you actually colour the map, estimate the minimum number of colours you will need.

______ colours

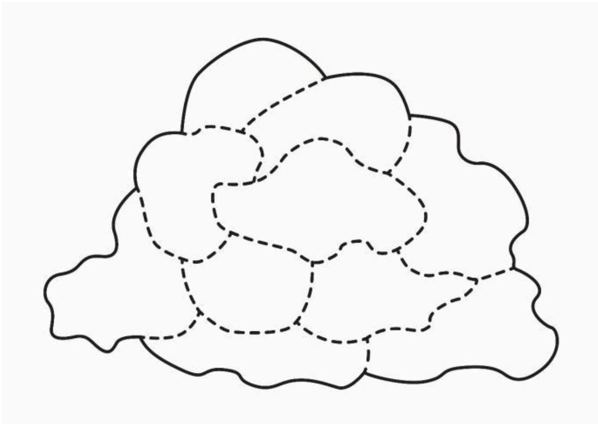


Figure 5

Q. How many colours did you actually need to colour the given map? ______

Compare your answer with the answers of your friends.

Q. Can you convert this map into a Network Diagram? Draw a Network Diagram representing this problem. Also colour the nodes of the Network Diagram. Remember that if a pair of nodes is connected by a link, that means they are neighbours on the map. So, they cannot have the same colour.

Task 3B: Colours of India

Q. Have you ever seen a coloured map of India? What have you noticed in it?

If you notice any map in the textbooks, atlas or on the internet, neighbouring regions (say states in a map of India) are always coloured in different colours.

Colouring the neighbourhood of a state.

Now look at the state of Manipur. From the map we can see that Manipur has three neighbours, namely Nagaland, Assam and Mizoram. We can also see that Assam and Nagaland are neighbours and so are Assam and Mizoram but Nagaland and Mizoram are not neighbours. How many colours do we need to colour the neighbourhood map of Manipur if we have to strictly follow the rule that any two neighbours must have different colours?

Remember how you coloured the imaginary kingdom in Task 3A? You can use the same strategies here too.

Q. What is the minimum number of colours you need to colour the neighbourhood map of Manipur? ___________________

Now choose another state of your choice and colour the neighbourhood of that state.

Q. How many colours will you need to colour the neighbouring states of the state you chose, such that any two states which are neighbours of each other have different colours or such that no two 'neighbouring' states have the same colours?

Task 4: **Network Diagrams and gathering of people**

In this task, we will see examples how the Network Diagram can help in modelling and solving various situations.

Imagine that you are at a gathering. And the organiser of this party wants it to be as colourful as possible. Thus, she gives the following instruction to all her guests: 'Talk to all the other guests you know and choose a dress of a colour different from their dresses'. As a result, we will have a situation where any two persons who know each other would have clothes of different colours.

1. **Draw Network Diagrams for all the following gathering situations and find out how many different coloured clothes would be needed in each of the parties**.

   A. If there are 4 people (A, B, C, and D) in the party. A and D know each other and nobody else anybody. What is the minimum number of different coloured clothes that you expect to see at this party??

   The gathering has 4 people.

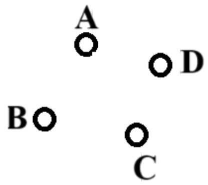
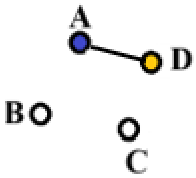


Figure 6



A and D know each other so we draw a link between them and they will wear different coloured clothes.

Figure 7

The others can wear either of the two colours.

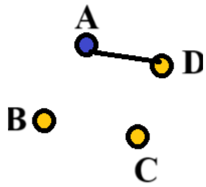


Figure 8

   B. If there are 4 people (A, B, C, and D) in the party and all people in the party know each other, how many different coloured clothes would be needed?

   C. If there are 5 people (A, B, C, D and E) in the party. A, B and C know each other and D and E know each other. How many different coloured clothes would be needed?

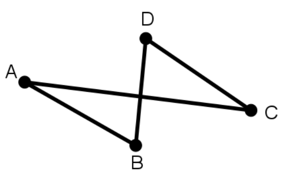
D. If there are 5 people (A, B, C, D and E) in the party. A and B know each other, B and C know each other, D and E know each other and C and E know each other. How many different coloured clothes would be needed?

E. If there are 5 people (A, B, C, D and E) in the party and all people in the party know each other, how many different coloured clothes would be needed?

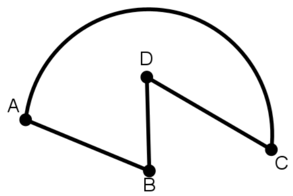
Task 5: Relation between nodes, links and colours

Let us see if there is any relation between the crossing and non-crossing links of a Network Diagram and minimum number of colours needed for that Network Diagram such that 'neighbouring' nodes have different colours.

Q. Look at the Network Diagram given here. You will notice that some of the links are crossing each other. Are the two Network Diagrams given below the same, i.e., do they represent the same situation?



Links are crossed



Links are not crossed

Figure 9

Q. Can you try to draw this Network Diagram such that no links cross each other?

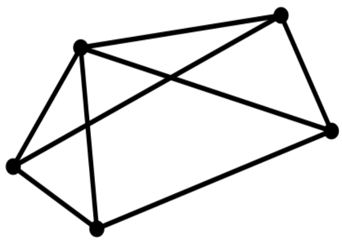


Figure 10

Q. Can you try to draw this Network Diagram such that no links cross each other?

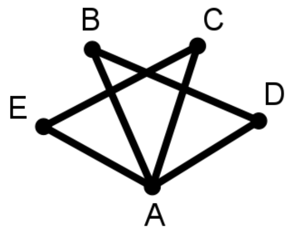


Figure 11

Set 5A: See the set of Network Diagrams given below. Fill the table.
(*'Neighbouring' nodes are nodes which have a link between them*)

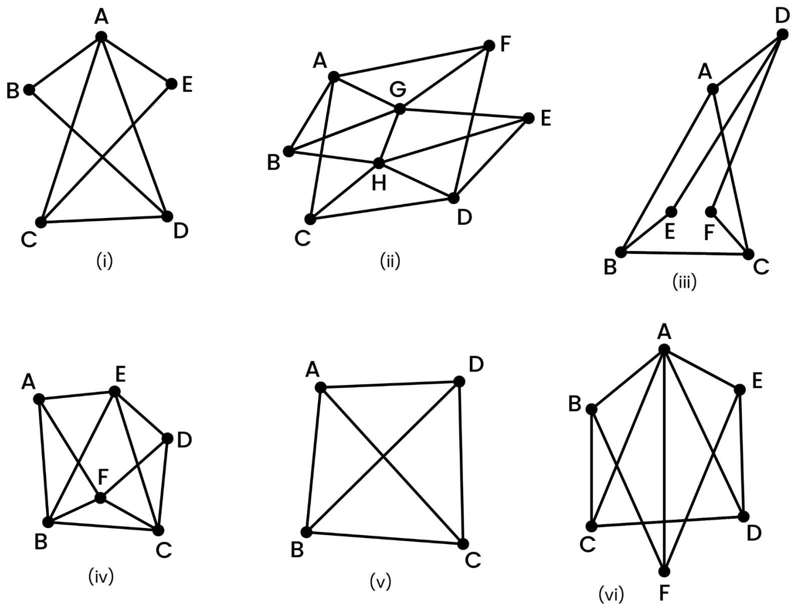


Figure 12

Network Diagram	**Total number of nodes**	**Minimum number of colours needed to colour the nodes such that 'neighbouring' nodes have different colours**	**Can you draw it without the links crossing each other?**
(i)			
(ii)			
(iii)			
(iv)			
(v)			
(vi)			

Table 1

Q. Did you notice any pattern in the table?

All the Network Diagrams given above can be drawn without their links crossing each other. Such Network Diagrams are examples of what is known as ‘Planar Network Diagrams’. Maps of regions drawn on paper can be always represented by planar Network Diagrams. Let us see if all Network Diagrams can be drawn such that their links do not cross.

Q. Can we find some Network Diagrams where it is impossible to draw it without the links crossing each other?

Set 5B: Fill the table for these Network Diagrams.
(*'Neighbouring' nodes are nodes which have a link between them*)

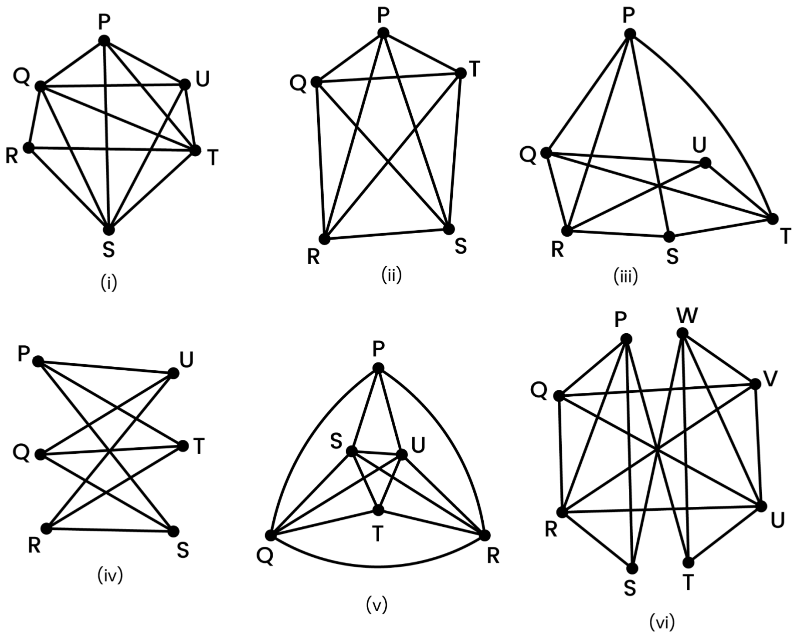


Figure 13

Network Diagram	Total number of nodes	Minimum number of colours needed to colour the nodes such that 'neighbouring' nodes have different colours	Can you draw it without the links crossing each other?
(i)			
(ii)			
(iii)			
(iv)			
(v)			
(vi)			

Table 2

Q. Did you notice any pattern in the table?

In the set 5B, we saw some Network Diagrams where drawing them without links crossing is impossible. Such Network Diagrams are called non-planar Network Diagrams as opposed to planar Network Diagrams we saw earlier. When you draw non-planar Network Diagrams on a 2-D surface (i.e., paper), the links will necessarily intersect each other and you cannot avoid the intersections of links by moving the nodes.

On the other hand, if you have real life examples of images on a 2-D surface (such as maps) and convert those images into Network Diagrams, then those will necessarily be planar Network Diagrams.

Q. Did you see any relation between Network Diagrams which can be drawn without the links crossing (i.e. planar Network Diagrams) and the number of colours that used to colour those Network Diagrams?
Note: The "relation" doesn't have to be a mathematical equation. You may describe your observations using words including relational terms such as "maximum", "minimum", "at least" (necessary), "at max" (sufficient), etc.