

Patterns in Square Numbers

Introduction: You must have calculated and studied squares of some whole numbers. In this unit we will study squares more carefully and try to find patterns in them. We will also go ahead and check if the pattern holds or that it does not.

Task 1

The following are the squares of numbers from 1 to 20.

What patterns do you see?

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400

-Ans- i) Pattern of odd and even.

ii) Ending no are 1, 4, 9, 6, 5. Digits are repeating in reverse order.

iii) Perfect squares when divided by 3 leave remainder either 1 or 0.

Only in 1st Column all numbers are odd

Task 2

Given below is a number chart. Circle the square numbers in the number chart. The first few are done for you. What patterns do you notice?

I	II	III	IV	V	VI	VII	VIII		I	II	III	IV	V	VI	VII	VIII
1	2	3	4	5	6	7	8		209	210	211	212	213	214	215	216
9	10	11	12	13	14	15	16		217	218	219	220	221	222	223	224
17	18	19	20	21	22	23	24		225	226	227	228	229	230	231	232
25	26	27	28	29	30	31	32		233	234	235	236	237	238	239	240
33	34	35	36	37	38	39	40		241	242	243	244	245	246	247	248
41	42	43	44	45	46	47	48		249	250	251	252	253	254	255	256
49	50	51	52	53	54	55	56		257	258	259	260	261	262	263	264
57	58	59	60	61	62	63	64		265	266	267	268	269	270	271	272
65	66	67	68	69	70	71	72		273	274	275	276	277	278	279	280
73	74	75	76	77	78	79	80		281	282	283	284	285	286	287	288
81	82	83	84	85	86	87	88		289	290	291	292	293	294	295	296
89	90	91	92	93	94	95	96		297	298	299	300	301	302	303	304
97	98	99	100	101	102	103	104		305	306	307	308	309	310	311	312
105	106	107	108	109	110	111	112		313	314	315	316	317	318	319	320
113	114	115	116	117	118	119	120		321	322	323	324	325	326	327	328
121	122	123	124	125	126	127	128		329	330	331	332	333	334	335	336
129	130	131	132	133	134	135	136		337	338	339	340	341	342	343	344
137	138	139	140	141	142	143	144		345	346	347	348	349	350	351	352
145	146	147	148	149	150	151	152		353	354	355	356	357	358	359	360
153	154	155	156	157	158	159	160		361	362	363	364	365	366	367	368
161	162	163	164	165	166	167	168		369	370	371	372	373	374	375	376
169	170	171	172	173	174	175	176		377	378	379	380	381	382	383	384
177	178	179	180	181	182	183	184		385	386	387	388	389	390	391	392
185	186	187	188	189	190	191	192		393	394	395	396	397	398	399	400
193	194	195	196	197	198	199	200		401	402	403	404	405	406	407	408
201	202	203	204	205	206	207	208		409	410	411	412	413	414	415	416

Task 3A1

Predict in which column the following will appear without calculating? Justify your answer. How do you know for sure?

- a) 75^2 b) 96^2 c) 122^2 d) 100000^2 e) 12346^2

Task 3A2

Is it possible that $(1234567)^2$ is in column 5? Why do you think so? *odd no. sq are in I column.*

Task 3A3

Prove at least 4 of the patterns that you came up with in Tasks 1 and 2. *sq. of odd no is odd*

Task 3B

Here is a square.



$$(2n+1)^2$$

$$(2n)^2 + 2(2n)(1) + (1)^2$$

$$4n^2 + 4n + 1$$

$2n+1 = \text{odd}$
 $2n = \text{even}$

We can add three more squares to make it into a 2×2 square.



2×2

Build on this to make a 3×3 square. How many squares did you have to add? *add 5*
Now use a 3×3 square to make a 4×4 square. How many squares did you add? *add 7*

Continue the process for a few more steps. What patterns do you observe? Is it related to any of the patterns that you noticed earlier? Can you prove the pattern?

$$n \times n \Rightarrow (n+1)(n+1)$$

$$3 \times 3 = 9 \Rightarrow (3+1)(3+1)$$

$$n^2 \Rightarrow (n+1)^2$$

Task 3C1

Plot the last digit of the square of a number against the number. What patterns do you see?

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$$

$$\frac{n^2 + 2n + 1 - n^2}{2n + 1}$$

$$2n + 1$$

$$n + n + 1$$

Task 3C2:

Notice that the pattern of last digits 1, 4, 9, 6, 5, 6, 9, 4, 1, 0 repeats.

Can you prove this pattern of repetition of last digits?

Task 3C3

Instead of the grid in Task 1 if we were to have a 10-column grid like the following, and circle the square numbers in the number chart. Can you guess in which columns these numbers would be?

Are there columns where there would be no circled numbers? *2, 3, 7, 8*

If you were to divide a perfect square by 10, what possible remainders would you get? *1, 4, 5, 6, 9*

I	II	III	IV	V	VI	VII	VIII	IX	X
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180

* Ist and IXth column are odd.

* In II, III, VI, VIII no sq. no. are there.