Precise and accurate measurements

Overview

There are four major requirements for scientific experimentation: procedural understanding (planning the experiment), recording observations(measurements), processing & analyzing those observations (calculations, graph, etc.) and working on errors for better results.

The core idea of this Learning Unit is to understand the following concepts: units of measurements, precision (how close is the observation compared to other observations made by the same instrument), accuracy (how close is the observation to the 'correct' or 'expected' observation), least count (never zero, not necessarily the minimum measurement of that instrument), and range (not just the upper limit, or a single measurement).

Minimum Time Required: 3 sessions of 40 minutes each (additional 40 minute session for the extended activity).

Type of Learning Unit: Laboratory/Classroom

Pre-requisite: Familiarity with common measuring instruments like ruler, thermometer,and brief knowledge of some common units of measurement, etc.

Materials Required:

Task 1: Eraser, sharpener, book, matchbox, other objects having cuboidal shape (such as a match box or a book).

Task 2: Easily available measuring instruments. geometry box (compass box), wrist-watch, stop watch, thermometers (laboratory and clinical), syringe (without needle), measuring cylinder, etc.

Task 3: Bob (any small yet heavy object can be used as a bob. eg. eraser), thread, stand, stop watch (mobile), ruler.

Task 5: Cardsheet paper/ cardboard, footscale, pencil, glue, sticky tape.

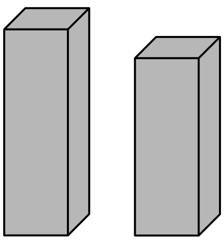
Introduction

One process which can be considered responsible for evolution of modern science, mathematics, and technology is developments in measurements. Any quantity for which a measurement system could be developed led to expanding our knowledge about the world and also led to new technologies.

Measurement, as simple a task as it may seem, gives us some values. These seemingly simple values carry a lot of information in them. In this Learning Unit, we are going to explore some details which are applicable to any measurement process through exploring measuring devices and the measurement data collected.

Task 1: Creating scales: Multiplying and Dividing

Imagine you do not have any measuring instrument handy with you. Can you compare the two blocks/objects in the diagram below?



Q1. How will you make the comparison? Can you say which one of them is taller and which one would weigh more? How did you make these conclusions?

You might get some information about the objects being compared, with reference to each other, but we don't know the exact height or weight of the objects. To have this information, we will have to measure these quantities using some measuring instrument.

Q2. Describe what you think 'measurement' is, in your own words. Also, what exactly is it that we do, when we measure any quantity?

We compare the unkown quantity using known objects/tools, assign a numerical value to the comparison and specify the quantity using a unit. It is such a simple everyday task, but we don't seem to appreciate its importance.

Now, suppose we want to measure the dimensions of a cuboidal object (i.e its length, breadth and height). For simplicity, we can try measuring the dimensions of a book. But you can also choose any cuboidal object of your choice eg. matchbox.

To measure its dimensions, we will need *something* to compare the lengths as before, such that the values obtained by comparing the cuboid object are multiples of that *something*. And this *something* could be anything that has a straight edge!

Q3. Can you think of some objects, other than a ruler, that we can use? What obvious feature does this object needs to have, to be able to measure the dimensions of the book?

To measure the length, students might intuitively think of using a ruler – which is a standardized instrument. The task suggests students to begin with some non-standard units, sense the challenges of comparing a measurement done with multiple non-standard units, and eventually understand the importance of standard units.

The feature needed to be able to measure is that the size (length) of the object used to measure must be smaller than the object being measured, atleast when its a non standard unit of measurement.

The object that you decide to use for measuring the dimensions of the cuboid, will be your reference for making a measuring instrument. You can use an eraser, a pen cap or any other suitable object. If you choose earser as reference object, any length that is equal to the length of the eraser will be 1 eraser unit. If twice then it'll be 2 eraser units and so on.

Now let's make a scale (on paper, cardboard, or any other material) using your eraser as a unit. You are free to design and develop this scale in any way you want, but like any other scale, this scale should also have markings on it for different lengths.

Q4. Where will you make the markings? Is there any other information one needs to make this scale?

Student's are expected to make markings on every eraser length and also may ask how long the scale should be.

In making a scale, what we are doing is making categories for lengths. Each marking on the scale defines a region and the dimensions of any object falling in the vicinity of that region, belongs to the region. For example, objects whose length is close to 1 eraser length will now belong to category of length = 1 eraser length. Similarly we can define another group of objects whose length can be defined to be 2 eraser length long. This is how the measuring will proceed. The categories that a scale creates are discretely finite and not infinite.

Once you are done making the scale, measure the dimensions of the cuboid object. One can also try using this scale to measure dimensions of different objects later.

Making a scale, especially for length measurements, depends upon the reference length used as a standard (in our case, an eraser length) and also the scale of the object being measured.
If the reference length is larger then it needs to be subdivided into certain number of equal parts, and if the reference length is small then one can use it multiple times repitetively.
i.e One either begins with reference range and then sub-divides it into smaller bits (essentially deciding the least count of the instrument) or start with the least amount of measurment i.e Least count and multiply it to get the reference range.
The standard reference for length is 1 meter from which other commercial length scales are made either by subdividing it into centimeters or multiplying it to get larger lenghts.
There's also a mathematical idea of 'n-secting a line segment' which would be useful for interested teachers to read about.

Objects	Measurements made using the new scale		
	Length	Breadth	Height

Table 1: Measuring cuboid objects using the new scale

Q5. Did you face any difficulties while taking the measurements? Were there some lengths you were not able to measure at all?

Here, students might mention the need for markings or smaller length, as the dimentions of the book may not coincide with the markings on their new scale. Lengths smaller than 1 unit of their scale cannot be measured, eg. The books thickness might be less than 1 eraser length.
Students can also think of way in which they want to subdivide each unit region between markings on their scale. An example would be folding the paper strip into halves to make half unit markings.

Q6. Can you use this scale to measure the length of your classroom? State your reasons.

Q7. Units are an important factor in any measurement. What were the units you used to measure the dimensions of the cuboidal object?

Students might intuitively say cm as they are measuring length, but point out to them that we haven't used a standard scale, but a non-standard one, thus the units would be, for example, 'eraser lengths' or 'eraser units'.

Now, based on the dimensions that you know describe the object that you have measured to other groups. Then you can discuss your findings within this group.

1) What did you discuss in groups?
2) Suppose each one of you decided on a different scale as your standard scale, would you be able to do and compare all types of measurements, or would you face some problems?
3) Now, based on the measurement, calculate the volume of cuboid. How does this value compare with others who have used the same object to measure?

Expect students to talk about how they might not be able to measure values below the least count of their instrument and how measuring longer distances could be a problem.

Task 2: Limits of a measuring instrument

In the previous task, we prepared our own length measuring instruments, i.e. the scale made out of eraser units, and got familiarised with some of its properties. These properties have some names designated to them. Let's see what they are called.

Consider any instrument, note down the value of the first marking and the last marking on it, that will give you the range of that instrument.

Here, we are not talking **just** about the maximum measurement of the instrument, but always from a certain minimum to a certain maximum, i.e., from ________ to________.

Q8. What is the range of the scale that you had made in Task 1?

___

Q9. With repeated measurements one can measure any higher value, e. g., to measure 42 cm, one can use the 30 cm ruler once again. So then, what is the range of the ruler?

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Q10. If the quantity we need to measure exceeds the range of the instrument, do you think we can always take repeated measurements to measure that physical quantity?

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Q11. Why does one need to know the range of an instrument?

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The range of an instrument specifies the extent to which a quantity can be measured. Though, like in rulers, one might be able to use the same ruler in repetition, it is not possible for quantities like temperature (by thermometer) or mass (using weight balance). Though it may seem like the range is infinite for measurements where repitition is possible, that is not the case as you might be limited for practical reasons. Teacher may also discuss how can one identify which instrument can and cannot be used to make a measurement in a particular range.

Students might notice in the first task that if the object is larger than the scale they made, then they can use repeatability to take measurement.

Another factor is the possibility of introducing errors due to repetition. If the teacher sees it fit, concept of errors can be discussed with students at this point.

Consider any instrument, and note down the smallest value it can measure, that will give you the least count of that instrument.

Least count of an instrument is the smallest change or difference in a value that can be reliably measured using that instrument, such that all the readings recorded are multiples of this value.The smallest value indicated on the scale of an instrument (mostly zero) is not the least count.

Q12. What do you think was the least count of the scale that you prepared in the task 1?

Q13. What do you think would be the least count of the 30 cm ruler we generally use?

Let the students explore an actual scale. There might be some students who might think that the least count of the 30 cm scale in 1 cm. Allow the students to discuss among themselves. Some of the scales may even have divisons in the 0.1 cm region (for a limited length), in that case the least count of that particular scale would be even less.

Q14. Why does one need to know the least count of an instrument?

Now, that you know the terms least count and range well, look at some common measuring instruments that you can find and fill the following table.

Sr. No.	Instrument	Measured quantity	Unit	Least Count	Range
1					
2					
3					
4					
5					
6					

Table 2: Range and least count of various measuring instruments

If the students don’t come up with instruments, the teacher can use the following list of instruments as a guide - measuring scale, stainless steel ruler, protractor, wrist-watch, stop-watch, laboratory thermometer, measuring cylinder.

A wrist watch measures the time in a cyclic order, i.e., the values get repeated after each cycle. Care should be taken while explaining the range of a digital watch as well as the analog watch. You might have to spend some time discussing the difference between a 24-hour clock and a 12-hour clock. Also, the difference in the range of a stop-watch and a wrist-watch need to be clarified.

One can also talk about different standard units of the same physical quanity like inches, cm, foot, etc.for lengths, Kelvin, Fahrenheit and Celcius for temperature, etc.

Task 3: Measuring time

Many instruments used to measure various physical quantity actually map that physical quanitity to length scale, thus transforming the measurement of that non-length quanity into a length measurement. Thus while measuring many physical quantities, we are essentially measuring length. These physical quantities are transformed into a corresponding calibrated lenght scale.

Using a tradational thermometer to measure temperature, it is calibrated to the rate of expansion of Mercury and what we measure is how much it has expanded on a length scale. Same is the case while measuring volumes of liquids using beakers or graduated cylinders. While using any analog scale to measure time, mass, etc., the concerned physical quanity is calibrated on a circular length scale.

Have you seen an old 'Grandfather clock', which has a suspended circular object , which goes back and forth continuously i.e. oscillates. Have you wondered how much time it might take for that suspended oscillating object, i.e a pendulum to complete one oscillation? Let's make a pendulum and try to observe that.

Take a string/thread and tie a small but heavy object (eg. an eraser, potato, etc.) at one of its end. Keep the length of the pendulum (from point of rotation to the centre of heavy object), say 50 cm. Tie the pendulum to a stand.

Now, you need to measure the time it takes for one oscillation. What measuring instrument will you use to measure the time period? How will you accurately record the starting and end of one oscillation of the pendulum?

Q15. Is there any similarity in how you did length measurement and time measurement?

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Students can be asked to come up with their own procedure to take the reading; they could measure time taken for just one oscillation or taking an average of time for 10 oscillations. Even though they would be using the same pendulum, there will be some variation in the time noted down by the students.

Q16. Was your reading the same as the other groups? Was there any difference? If yes, what could be the reason? What can be done to minimize this difference?

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To have more reliable measures, an average of multiple readings can be taken.

An interesting observation: Note that even with very high precision measurements, the measure value will always with a rational number times the basic unit. One can never get an irrational number times the basic unit/least count from a physical measurement!

Task 4: Repeating a measurement and variations obtained

Most of the variations that occur during measurements can have either of the following two kinds of origin:

- Environmental variation: This kind of variation can happen due to changes in the surrounding environment which can effect the quantity being measured, the scale used for measruement, even both. Changes like temperature, pressure, presense of electric and magnetic fields can cause variation in the nature of matter itself. Measurements taken dusing different times of the day or in different seasons can bear this variation.

- Observational variation: The measurement techniques employed by the person taking the measruement , using the same instrument, can even lead to variations. For eg. Accounting for parallax while making measements.

In task 3, we noted down the time period for the pendulum we made. For reference, some more readings taken by student 1 and student 2, using a pendulum with same length of 50 cm, have been given below. Add your readings to this table in the last column.

Now, let's take a look at the data given below and try to analyze it.

Time taken for 1 oscillation	Student 1 (in s)	Student 2 (in s)	Your (in s)
t_1	1.40	1.44	
t_2	1.40	1.40	
t_3	1.40	1.42	
t_4	1.40	1.40	
t_5	1.40	1.43	

Table 3: Experimental data for time period of a pendulum

Q17. From the table, what can you say by looking at the values? Who do you think amongst students 1 and 2 performed the experiment well and why? How does your data compare with theirs?

____

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The first thing one will notice on looking at the data is the consistency i.e. precision in the readings taken by student 1 and the scatter in the readings taken by student 2, which gives the feeling that student 1’s reading might be correct. But precise reading might not always be accurate.

Teacher may suggest students to find the acceleration due to gravity ‘g’. Based on this mathematical procedure, again prompt for whose experiment is more correct? After ‘g’ is calculated, one will realize that even though Student 2’s readings seemed scattered it has better accuracy than student 1.

In the pendulum example, you might have noticed many words used by peers including correct, precise, perfect, accurate, to the point. Let us consider some more situations before we explore what meaning these words have

Q18. You buy 500 g potatoes or tomatoes. Then you are buying a silver ornament which is weighing 20 g. In which case you would be extra careful about correct weight?

____

____

____

Q19. You need to cut a 2 m long plank of wood to make a sitting bench in a park, versus you need to cut a 2 m plank to make a door which need to fit in already fixed frame in a wall. In which case you need to be extra careful in length measurement?

____

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Students may say a small deviation in weight measurment may cost extra money or variation in cutting length will not fit in the frame accurately. Extra care in measruement of the later examples in Q18(silver) and Q19(door) is crucial. These examples reveal importance of why we need to think carefully about the measruement.

Variations in multiple measurements of same object

Q.20 A lady purchased a *payal* (a feet anklet) made of silver from a village goldsmith, which weighted 21.3 g. She paid to the goldsmith for that mass of silver. But after coming out of goldsmith's shop, she double checked the mass from a different shop in the village where it weighed 21.4 g. She then went to her home in a citywhere goldsmith in another shop weighed the payal and the mass was found to be 22.1 g.

Why do you think the mass of the same ornament was different in three different shops. How can one decide what is the correct value of mass?

The variation in multiple measurements of same object is understood using two ideas: precision and accuracy.

A classic way of demonstrating the difference between precision and accuracy is with a dartboard. Think of the center (also known as bulls-eye) of a dartboard as the true value. The closer darts land to the center (the desired or true value), the more accurate they are.

Teacher may use the video that explains precision and accuracy (or summarise the content).
https://www.ted.com/talks/
matt_anticole_what_s_the_difference_between_accuracy_and_precision?language=en

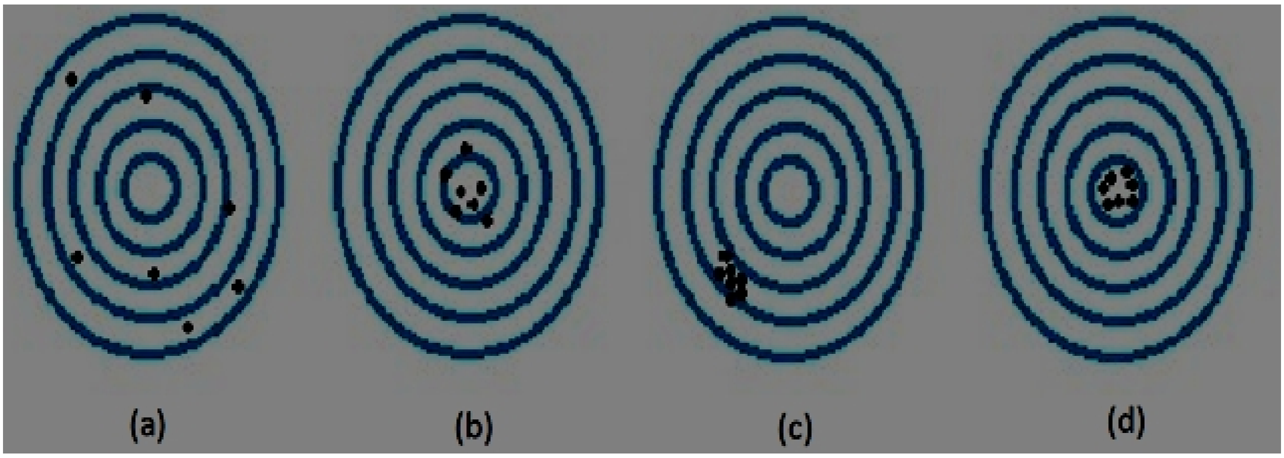

Figure 1: A classic example of using a dartboard to explain concepts of accuracy and precision

Q21. **So, if the target is always hit at the same place all the time, the player is said to be precise.**

Which of the cases above (among a-d) seem more precise to you, and which less?

Q22. But the player is only accurate when those precise hits are at the center. Now, which one is more accurate and which less?

• (a) If the darts are neither close to the bulls-eye, nor close to each other, there is neither accuracy, nor precision .

• (b) If the darts are all about an equal distance from and spaced equally around the bulls-eye there is mathematical accuracy because the average of the darts is in the bulls-eye. This represents data that is accurate, but not precise. However, if you were actually playing darts this would not count as a bulls-eye!

• (c) If all of the darts land very close together, but far from the bulls-eye, there is precision, but not accuracy.

• (d) If the darts land close to the bulls-eye and close together, there is both accuracy and precision .

In the dart board game, we can easily talk about accuracy, because we know **where** we have to hit to get the best result. But during experiments or real life measurements, one doesn't really know what the bull's eye is! Similar to the simple pendulum experiment. The aim is to find the time period for 1 oscillation. But one can never be sure what the correct answer for that is! And many factors affect the result of any experiment and deviate it from the true value causing errors.

Thus, the goal of an experimenter is not getting accurate value, but to think about all the factors that can cause errors and trying to reduce them as much as possible to get the best result out of any setup. And if the same experiment is repeated at different time or place or by different people and the result are same, then that value is accepted as the correct value. But over time, if someone realises the existence of some other error causing factor and changes the experiment to reduce that error, a new result will be obtained. This result if found reproducible, be considered as the new correct result.

Students might say that the result can be derived by using formulas. They should be explained that, even in formulas one uses some measured values to get the result and thus obtained result is still not accurate. And precedence is given to experimentally found value than the derived result.

Task 5: Need for high and low precision in measurement

Increasing precision of a measurement requires improvement in technologies and measurement methods. It may also increase costs of measurements/measuring equipments. In many case low precision measurements are also okay. Suggest which order of precision would be needed for following measurements:

1) Amount of water to be given to a healthty person for drinking: ±100 mL, ±10 mL, ±1 mL, ±0.1 mL

2) Carrots to be purchased for one family's meal: ±100 g, ±10 g, ±1 g, ±0.1 g

3) Height of wood pieces to be cut for a chair: ±10 cm, ±1cm, ±0.1 cm, 0.01 cm

4) A pair of gold earrings: ±1 g, ±0.1 g, ±0.01 g, 0.001 g

5) Time taken by a train to travel from one station to next station: ± 10 min, ± 1 min, ±10 sec, ±1 sec

6) Length of inside box of a matchbox to fit inside outer box : ± 0.5 cm, ±0.2 cm, ±0.1 cm, ±0.02 cm

Task 6 Dividing a unit into subunits- The Vernier's method (extended activity)

Vernier calliper is a useful instrument for making precise length measurements smaller than 0.1 cm. The least count of the vernier scale can be 0.01 cm depending on the making of the scales.

The Vernier calliper has a main scale with markings graduated at a distance of 0.1 cm or 0.5 cm, which is fixed and does not move. A movable scale called the 'Vernier' scale is attached on the main scale. Movable scale has markings which are slightly shifted relative to the markings on the main scale by some distance depending upon the precision required from the instrument. To start making this instrument, you would need five pieces of the following shapes. Choose total length of the scale (say 10 cm or 15 cm or 20 cm).

Procedure:

Choose a length (end-to-end) for the vernier caliper (10 cm/15 cm/20 cm) & accordinly draw the parts labelled A and B on a card board/card sheet/paper. Cut them along the outer border and out of the cardsheet carefully. A will be our main scale while B will serve as the vernier scale.

After cutting out the shapes, lay part A over part B, such that the dashed lines of both the parts are aligned with each other.

Along the bottom horizontal line of part A, make markings every 1 cm, where the dashed line marks the '0' of the scale. Also add length values as 1 cm, 2 cm, 3 cm and so on on the scale.

Similarly, along the bottom horizontal line of part B (with the dashed line marking the '0' of the scale, along the edge of the main scale drawn on part A, when it is kept over part B), make markings every 0.9 cm with a range of 9 cm numbering each of them as 1(at 0.9 cm), 2(1.8 cm),....

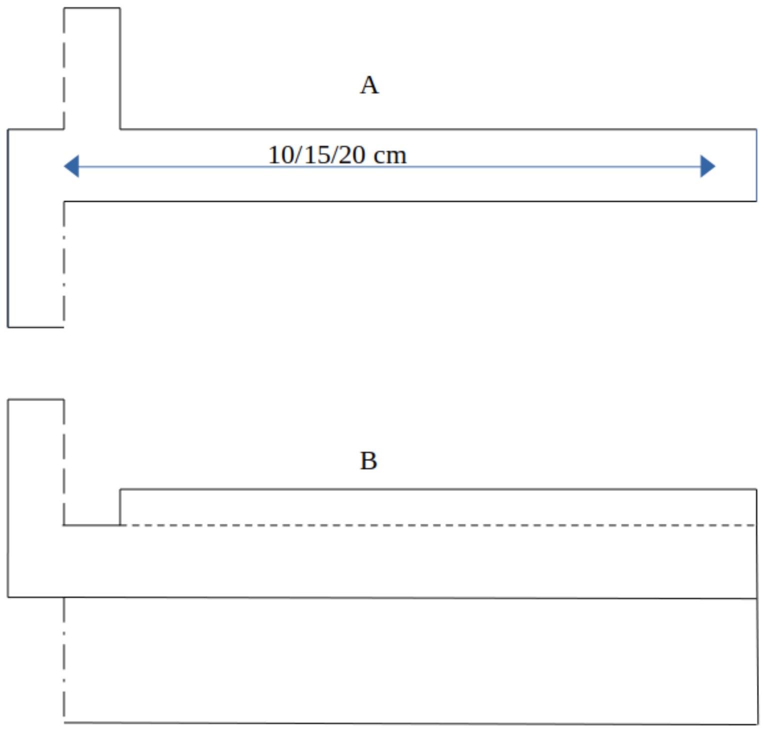

Figure 2: Outline shape for making a Vernier Caliper

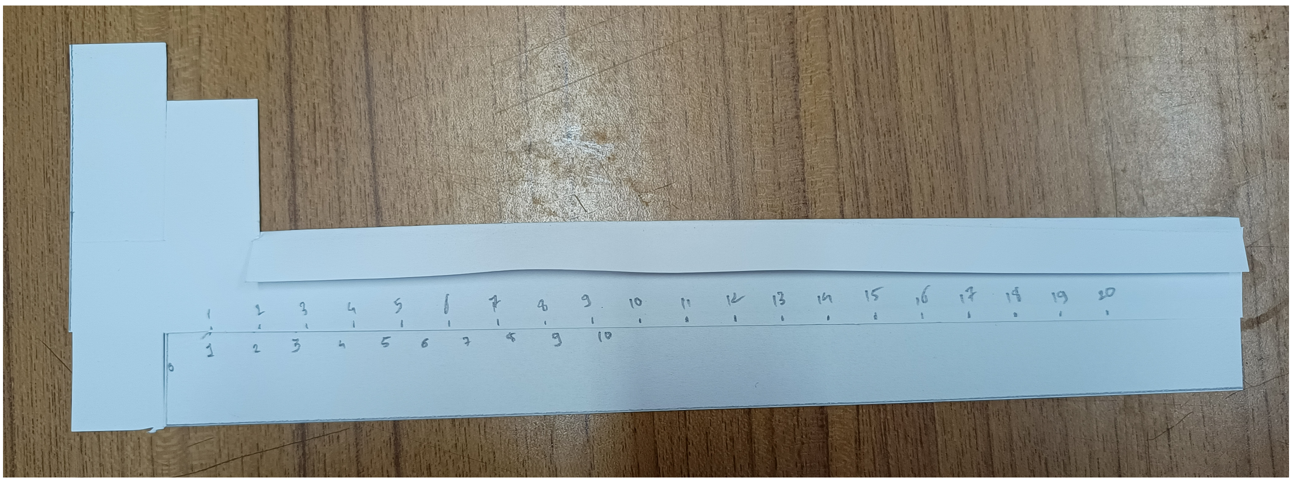

Image 1: An assembled Vernier Caliper made on paper

When the instrument(calipers) is closed or making zero measurement, the "0" marking on the main scale as well as the vernier scale exactly coincide with each other. The "0"th marking on the vernier scale is essentially the starting point of the vernier scale.

You can notice that the first mark on the vernier calliper is 1/10th cm or 0.1 cm short(to the left) of the first mark on the main scale. Similarly, the second marking on the vernier scale will be 2/10th cm short(to the left) of the second mark on the main scale. Similarly, the 3rd is 3/10th cm short and 8th is 8/10th cm short of the 3rd and 8th marking on main scale respectively. The 10th marking on the vernier will be 10mm or 1 cm short of the corresponding mark on the main scale and aligns 1cm to the left of the 10 cm mark on main scale i.e., coincides with the 9 cm marking on the main scale. Therefore, **there are 10 divisions on the vernier scale for 9 divisions on the main scale, thus separating them by an interval of 0.1 cm.**

Let's understand how this small displacement/misalignment of the markings on the two scales helps us to make fine measurements. Suppose we need to make a length measurement of an eraser of 3.5 cm in length. We need to slide the calliper to the desired length. The zero on the vernier scale will be ahead of the 3 cm mark on the main scale, which will be the main scale reading. To complete our measurement, we need to know the gap between the 3 cm on on the main scale and the coinciding division on the Vernier scale. The amount by which the zero on the vernier scale is ahead of the 3 cm mark is 0.5 cm, which 5 mm.

The first mark on the vernier scale which was earlier short by 0.1 cm will now be ahead by a net distance of more than 3 cm from the zero mark on the main scale and will not align with any marking on main scale. Similarly, the second and third mark on the vernier scale will move ahead and won't align with any marking on main scale.

However, the fifth marking on the vernier scale which was 0.5 cm short will be 0.5 cm ahead and coincide exactly with the marking on the main scale. The coincidence can be visually seen and noted by an observer. Thus, the total length of the matchbox will be 3 cm (which is 3 main scale division) + 0.5 cm (Vernier scale division) = 3.5 cm.

Thus, the instrument with a main scale least count of 1 cm or 0.1 mm along with a vernier scale could measure upto 0.1 cm which is its least count. It is thus only logical to have a vernier scale which has even more closely spaced graduations to increase the accuracy.

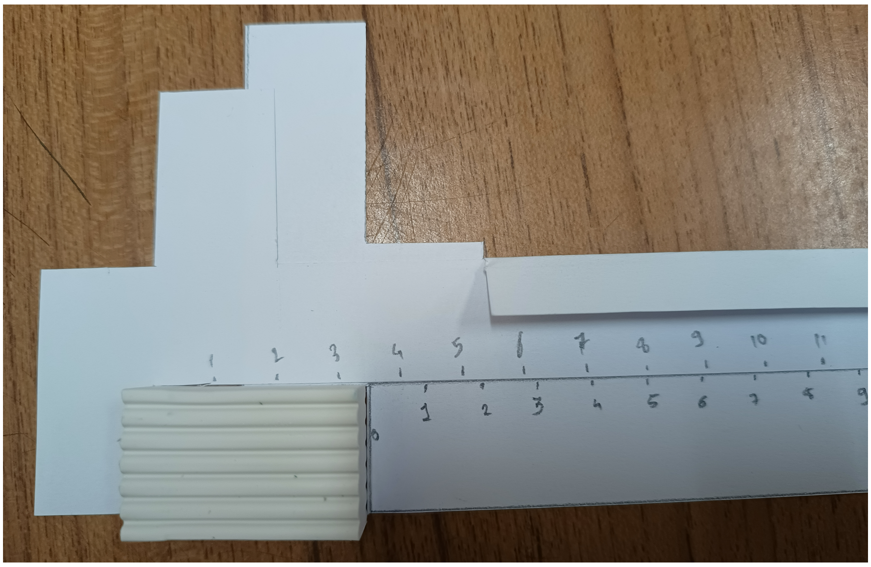

Image 2: Measuring length of an eraser using Vernier Caliper

In the above image, the 5th division on the Vernier scale is seen to be coinciding, there is no gap here. The gap of our instrument (where the eraser's length fits) is the gap between 3 main scale divison and 5 Vernier scale divison, i.e. 5 mm or 0.5 cm.