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Topic : Exploring patterns in square numbers

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Exploring Patterns in Square Numbers

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MATHS

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Acknowledgement

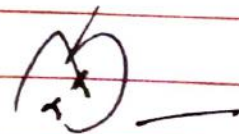
I would like to express my special thanks of gratitude to my mathematics teacher, **Mr S.N. Das**, who gave me this wonderful topic on "Exploring patterns in square numbers". This helped me a lot in doing Research and I came to know about so many new things. I am really thankful to them.

Secondly, I would also like to thanks my parents and friends who provided me with the necessary things for it.

Thank You!

Signature of Student : **Bhabani Sankar Sarangi**

Signature of Guide teacher:



Overview :-

Students explore patterns in square numbers arranged in different ways. The unit aims to introduce students to some of basic practices and way of thinking of mathematics as a discipline - for example, reasoning, conjecturing, proving, problem solving, representing, seeing, and using connections, generalizing, abstracting and communicating precisely.

Learning objectives :-

- (i) observing and coming up with patterns
- (ii) Expressing a pattern algebraically
- (iii) Extending and verifying a pattern
- (iv) Visualizing and proving a pattern
- (v) Understanding the difference between verification and proof, difference between theorem and conjecture
- (vi) Exploring relationships between statements

Materials Required :- Graph paper, Printed number grids.

Prerequisites :- Square numbers, Algebraic Identities, Operations on Polynomials

Task 1

The following are the squares of numbers from 1 to 20.

Task 2

It is obvious to see that the numbers with circles appear only in columns I, IV and VIII. There is a regularity in the gaps between the circled numbers, which may not be so obvious.

8. In Column I, the gaps increase by 1 row each time, the gaps are as follows : 0, 1, 2, ...
9. In Column IV, the gaps increase by 4 rows each time, the gaps are as follows : 3, 7, 11, ...
10. In Column VIII, the gaps increase by 4 rows each time, the gaps are as follows : 5, 9, 13, ...

Some patterns :-

11. The squares of all odd numbers are in Column I.
12. The squares of even numbers are in columns IV or VIII.
13. The squares of multiples of 4 are in Column VIII.
14. The squares of even numbers that are not multiples of 4 are in Column IV.
15. All the squares of numbers of type $8n+1$, $8n+3$, $8n+5$ and $8n+7$ are in the first column, $8n+2$, $8n+6$ are in the fourth column and $8n+4$ and $8n$ are in the eighth column.

Task 2

I	II	III	IV	V	VI	VII	VIII		I	II	III	IV	V	VI	VII	VIII
①	2	3	④	5	6	7	8		209	210	211	212	213	214	215	216
⑨	10	11	12	13	14	15	⑩		217	218	219	220	221	222	223	224
17	18	19	20	21	22	23	24		⑪	226	227	228	229	230	231	232
⑫	26	27	28	29	30	31	32		233	234	235	236	237	238	239	240
33	34	35	⑬	37	38	39	40		241	242	243	244	245	246	247	248
41	42	43	44	45	46	47	48		249	250	251	252	253	254	255	⑭
⑮	50	51	52	53	54	55	56		257	258	259	260	261	262	263	264
57	58	59	60	61	62	63	⑯		265	266	267	268	269	270	271	272
65	66	67	68	69	70	71	72		273	274	275	276	277	278	279	280
73	74	75	76	77	78	79	80		281	282	283	284	285	286	287	288
⑰	82	83	84	85	86	87	88		⑲	290	291	292	293	294	295	296
89	90	91	92	93	94	95	96		297	298	299	300	301	302	303	304
97	98	99	⑳	101	102	103	104		305	306	307	308	309	310	311	312
105	106	107	108	109	110	111	112		313	314	315	316	317	318	319	320
113	114	115	116	117	118	119	120		321	322	323	⑳	325	326	327	328
⑳	122	123	124	125	126	127	128		329	330	331	332	333	334	335	336
129	130	131	132	133	134	135	136		337	338	339	340	341	342	343	344
137	138	139	140	141	142	143	㉑		345	346	347	348	349	350	351	352
145	146	147	148	149	150	151	152		353	354	355	356	357	358	359	360
153	154	155	156	157	158	159	160		㉒	362	363	364	365	366	367	368
161	162	163	164	165	166	167	168		369	370	371	372	373	374	375	376
㉓	170	171	172	173	174	175	176		377	378	379	380	381	382	383	384
177	178	179	180	181	182	183	184		385	386	387	388	389	390	391	392
185	186	187	188	189	190	191	192		393	394	395	396	397	398	399	㉔
193	194	195	㉕	197	198	199	200		401	402	403	404	405	406	407	408
201	202	203	204	205	206	207	208		409	410	411	412	413	414	415	416

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Topic : Exploring patterns in square numbersTask 3A1

- (a) 75 being an odd number, 75^2 will be in the first column and leaves a remainder 1 when divided by 8. (Using Statement (11) and its reformulations)
- (b) 96 being an even number that is divisible by 4, its square will be in column VIII. (Using Statement (13) and its reformulations)
- (c) 122 being an even number that is not divisible by 4, its square will be in column IV. (Using Statement (14) and its reformulations)
- (d) 100000^2 - Column VIII (Using Statement (13) and its reformulations)
- (e) 12346^2 - Column IV (Using Statement (14) and its reformulations)

Task 3A2

The square of $(1234567)^2$ is not in column 5 as we know that the square of all odd numbers will be in column I.

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Topic : Exploring patterns in square numbersTask 3A3

Proof of some of the patterns from task 1 and 2:

Task 1

Pattern 1 : "The squares of all odd numbers are odd."

Any odd number can be written in the form $2n+1$, where n is a whole number.

$$\begin{aligned}
 (2n+1)^2 &= 4n^2 + 4n + 1 \\
 &= 2(2n^2 + 2n) + 1 \\
 &= 2m+1, \text{ where } m = 2n^2 + 2n \text{ is a whole number.} \\
 &\quad \text{(Hence proved).}
 \end{aligned}$$

Pattern 2 : "The difference between successive square numbers is the sequence of odd number."

Let the 2 numbers be n and $n+1$, where n is a natural number.

$$\begin{aligned}
 (n+1)^2 - n^2 &= \cancel{n^2} + 2n + 1 - \cancel{n^2} \\
 &= 2n + 1
 \end{aligned}$$

 $2n+1$ is always an odd number for any natural number, n .

(Hence proved).

Ques

Ans - 6

The following numbers are square numbers

1. The squares of all odd numbers are in column 1 or 3 of the type $4n+1$.
 2. All numbers in column 2 or 4 are in the form $4n+2$ or $4n+3$ which are not square numbers.

$$\begin{aligned}(n+1)^2 &= n^2 + 2n + 1 \\ &= 4\left(\frac{n^2}{4} + n\right) + 1 \\ &= 4\left(\frac{n^2}{4} + n\right) + 1\end{aligned}$$

n is odd $\Rightarrow$ it always leaves a remainder of 1 when divided by 4.

$$4n+1 = 4\left(\frac{n^2}{4} + n\right) + 1 = 4\left(\frac{n^2}{4} + n\right) + 1$$

hence proved

2. The squares of multiple of 2 are in column 2 or 4 i.e. multiple of 4. As the numbers in column 2 or 4 are in the form $4n+2$ or $4n+3$ which are not square numbers.

$$\begin{aligned}(2n)^2 &= 4n^2 \\ &= 4\left(\frac{n^2}{1}\right) \\ &= 4n^2\end{aligned}$$

hence proved

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Topic : Exploring patterns in square numbersTask 2

Pattern 1: "The squares of all odd numbers are in column I or of the type $8m+1$."

Any odd number can be written in the form $2n+1$, where n is a whole number.

$$\begin{aligned}(2n+1)^2 &= 4n^2 + 4n + 1 \\ &= 4(n^2 + n) + 1 \\ &= 4n(n+1) + 1\end{aligned}$$

Now $n(n+1)$ is always an even number, because either n is even or $n+1$ is even.

$$(2n+1)^2 = 4n(n+1) + 1 = 8m + 1, \text{ where } m = \frac{n(n+1)}{2}$$

(Hence proved)

Pattern 2: "The squares of multiples of 4 are in column VIII or are multiples of 8"

Any multiple of 4 can be written in the form $4n$, where n is a whole number.

$$\begin{aligned}(4n)^2 &= 16n^2 \\ &= 8(2n^2) \\ &= 8m, \text{ where } m = 2n^2\end{aligned}$$

(Hence proved)

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Topic : Exploring patterns in square numbersTask 3B

This task is a visualisation of the fact that the difference between squares of consecutive numbers is an odd number.

To build 3×3 square on a 2×2 square, the outer edge of orange squares needs to be added. Similarly, to build higher order squares, one should keep on adding the outer edge.

The outer edge that is to be added to a 3×3 square to make it a 4×4 square consists of $3+4=7$ squares.

Similarly, the outer edge that needs to be added to a 4×4 square to make it a 5×5 square consists of $4+5=9$ squares.

In general, the outer edge that needs to be added to a $n \times n$ square to make it an $(n+1) \times (n+1)$ square will have $n+n+1 = 2n+1$ squares, which is $(n+1)^{\text{th}}$ odd number. This could be proved algebraically as well.

The difference between two consecutive squares can be written as 0, $(n+1)^2 - n^2 = 2n+1$

Task 38



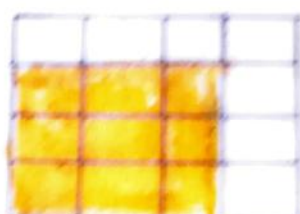
2

$$1 + 3 = 4$$



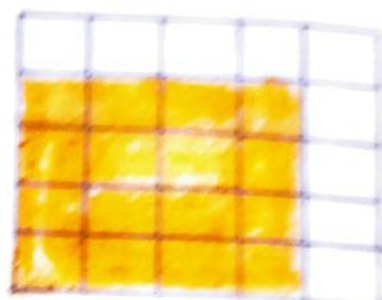
3

$$4 + 5 = 9$$



4

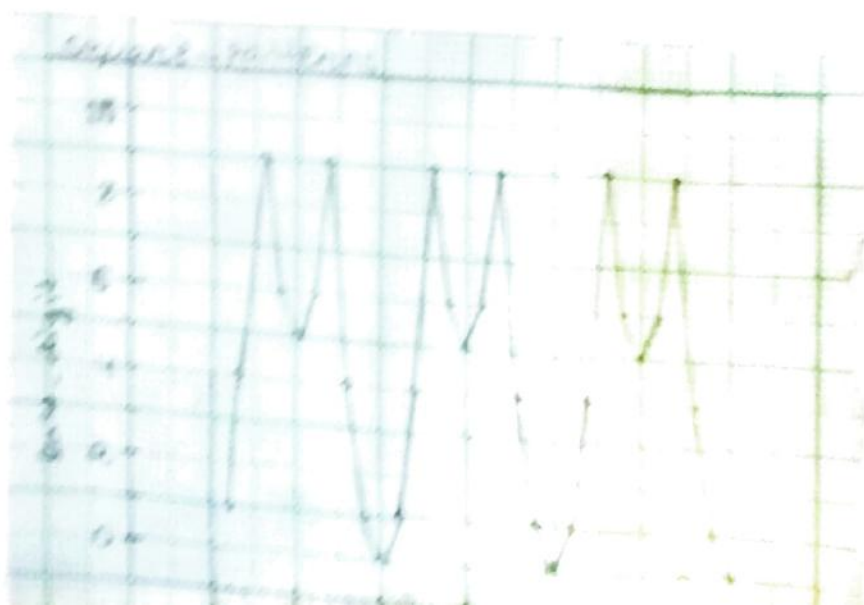
$$9 + 7 = 16$$



5

$$16 + 9 = 25$$

Task 39



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Topic : Exploring patterns in square numbers

sk 3C1

The graph helps to see the repeating pattern and symmetry. Notice the repeating pattern and the symmetry about the lines at $x = \text{multiples of } 5$.

sk 3C2

The units digit of any number is the remainder obtained on dividing the number by 10.

Any number can be written in the form $10n + K$

$$(10n + K)^2 = 100n^2 + 20nK + K^2$$

The units digit of $100n^2 + 20nK$ is 0 as it is a multiple of 10.

∴, the units digit of $100n^2 + 20nK + K^2$ is the same as the last digit of K^2 .

The units digit of K^2 and $(10n + K)^2$ are identical.

Thus, there is a repeating pattern.

Ex:- The repeating unit is symmetric about multiples of 5, which means that the last digits of $5n + K$ and $5n - K$ are the same. This can be proved as follows.

$$(5n + K)^2 = 25n^2 + 10nK + K^2$$

$$(5n - K)^2 = 25n^2 - 10nK + K^2$$

$$(5n + K)^2 - (5n - K)^2 = 10nK$$

The difference between these two numbers is $10nK$, which has a zero at the units place and therefore

Task 3C3

I	II	III	IV	V	VI	VII	VIII	IX	X
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180

they both have the same units digit.
 Also, the last digit of k^2 and $(10n - k)^2$ are identical.

Task 3C3

Task 2 was based on the remainders that a perfect square would leave when divided by 8, whereas Task 3C was about the remainders that a perfect square would leave when divided by 10. Both are essentially similar.

The remainder when divided by 10 happens to be the last digit as well.

Task 3C4

When a perfect square is divided by 3 one cannot get a remainder of 2.

Task 4

Multiple reasons/arguments may be articulated, for example -

(a) The last digits of a perfect square follow the pattern 1, 4, 9, 6, 5, 6, 9, 4, 1, 0. The digit 2, the last digit does not belong to this pattern. Therefore, it is not a perfect square.

(b) A perfect square leaves a remainder 1, 4 or 0 when divided by 8. 122 leaves a remainder of 2 and therefore it is not a perfect square.

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Topic : Exploring patterns in square numbers

(c) 11^2 is 121, 12^2 is 144. So, all numbers between these two numbers will have their square roots between 11 and 12 and are therefore not perfect squares.

But, these arguments may not work for some numbers. 356 has the last digit 6 and leaves a remainder 4 when divided by 8. So, arguments of the form (a) and (b) will not work in this case. However, an argument can be built up along the lines of (c). Other reasons like

(d) A perfect square leaves a remainder 1 or 0 when divided by 3, but 356 leaves a remainder 2. Hence it is not a perfect square can also come up.

(e) A perfect square if it has the units digit 0, must have the hundreds digit 0 as well. This is not the case with 340 and therefore it is not a perfect square.

The number 425 is in column I, but not a perfect square. There are many more numbers in column I which are not perfect squares.

Task 3C4

I	II	III
1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21
22	23	24
25	26	27
28	29	30
31	32	33
34	35	36
37	38	39
40	41	42
43	44	45
46	47	48
49	50	51
52	53	54
55	56	57
58	59	60

I	II	III
61	62	63
64	65	66
67	68	69
70	71	72
73	74	75
76	77	78
79	80	81
82	83	84
85	86	87
88	89	90
91	92	93
94	95	96
97	98	99
100	101	102
103	104	105
106	107	108
109	110	111
112	113	114
115	116	117
118	119	120

I	II	III
121	122	123
124	125	126
127	128	129
130	131	132
133	134	135
136	137	138
139	140	141
142	143	144
145	146	147
148	149	150
151	152	153
154	155	156
157	158	159
160	161	162
163	164	165
166	167	168
169	170	171
172	173	174
175	176	177
178	179	180

I	II	III
181	182	183
184	185	186
187	188	189
190	191	192
193	194	195
196	197	198
199	200	201
202	203	204
205	206	207
208	209	210
211	212	213
214	215	216
217	218	219
220	221	222
223	224	225
226	227	228
229	230	231
232	233	234
235	236	237
238	239	240

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Topic : Exploring patterns in square numbers.

using the language of mathematics, last digit being one of 1, 4, 9, 6, 5 or 0, or leaving remainders 0, 1 or 4 when divided by 8 are necessary conditions for a number to be a square.

consider the number 1000:

It has the last digit 0, digit in the hundreds place is also 0, leaves a remainder 0 when divided by 8, 1 when divided by 3 and yet is not a perfect square.

References:-

<https://nrich.maths.org/7117>

<https://nrich.maths.org/2280>

<https://nrich.maths.org/4350>

Exploring Patterns in Square numbers

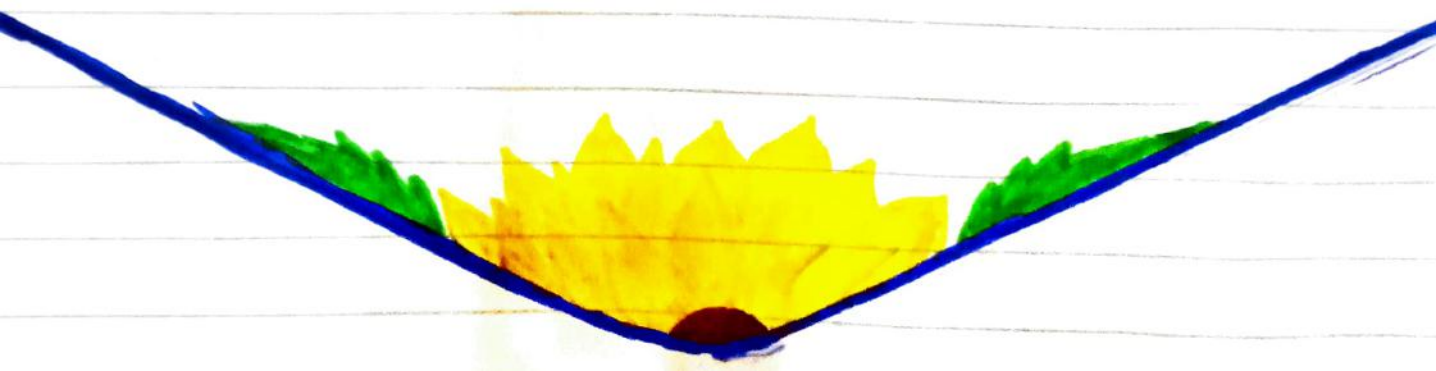
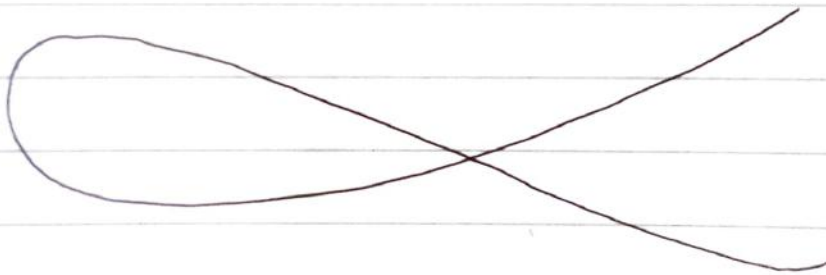
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Exploring Patterns In Square numbers

Task - 1 :-

Number	①	②	③	④	⑤	⑥	⑦	⑧	⑨	10	11	12	13	14	15	16	17	18	19	20
Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400

The squares of numbers from 1 to 20, We observe that, (with odd no.s)

$$\rightarrow (1+3)1 = 4 = \text{Square of } 2 \quad (\because 1 < \textcircled{2} < 3)$$

$$\rightarrow (3+5)2 = 16 = \text{Square of } 4 \quad (\because 3 < \textcircled{4} < 5)$$

$$\rightarrow (5+7)3 = 36 = \text{Square of } 6 \quad (\because 5 < \textcircled{6} < 7)$$

$$\rightarrow (7+9)4 = 64 = \text{Square of } 8 \quad (\because 7 < \textcircled{8} < 9)$$

----- etc.

Like this, we can calculate. So, ~~for~~ we observed this pattern.

Task - 2 :-

I	II	III	IV	V	VI	VII	VIII
①	—	—	④	—	—	—	①6
⑨	—	—	③6	—	—	—	⑥4
②5	—	—	①00	—	—	—	①44
④9	—	—	①96	—	—	—	②56
⑧1	—	—	③24	—	—	—	④00
①21	—	—		—	—	—	
①69	—	—		—	—	—	
②25	—	—		—	—	—	

Teacher's Signature & Date :

Here we observed that,

i) In 1st column

The square of 1, 3, 5, 7, 9, 11, 13, 15, 17, 19 ---
are present.

ii) Then ~~there~~ there is not any square no.
in 2nd & 3rd column.

iii) In 4th column

The square of 2, 6, 10, 14 ---
are present.

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
 (2x1) (2x3) (2x5) (2x7)

iv) Then, again there is not any square no.
in 5th, 6th & 7th column.

v) In 8th column

The square of 4, 8, 12, 16, 20 ---
are present.

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
 (4x1) (4x2) (4x3) (4x4) (4x5)

So, here we observed this pattern.

Task 3A1:-

- (a) $75^2 \rightarrow$ It is present in 1st column.
 (b) $96^2 \rightarrow$ It is present in 8th column.
 (c) $122^2 \rightarrow$ It is present in 8th column.
 (d) $100000^2 \rightarrow$ It is present in 4th column.
 (e) $12346^2 \rightarrow$ It is present in 4th column.

Task 3A2

No, it is not possible that $(1234567)^2$ is in Column-5. Because as we see & concluded that there is not any square number present in Column-5.

Task-3A3

From task-1 & task-2

We proved that,

- i) From 1 to any number, we can find a pattern like this:-

No. $\rightarrow$	①	2	③	4	⑤	6	⑦	8	⑨
Sq. no. $\rightarrow$		4		16		36		64	
	[(3+1) $\times$ 1]		[(3+5) $\times$ 2]		[(5+7) $\times$ 3]		[(7+9) $\times$ 4]		

- ii) In 1st column, there is the number 1, 3, 5, 7, 9, 11, 13, 15, 17, 19 ---- & square no.s are present.

iii) In 4th column we observed the square of $(2 \times 1) = 2$, $(2 \times 3) = 6$, $(2 \times 5) = 10$, $(2 \times 7) = 14$ are present.

iv) In 8th column, we observed the pattern that the square of $(4 \times 1) = 4$, $(4 \times 2) = 8$, $(4 \times 3) = 12$, $(4 \times 4) = 16$, $(4 \times 5) = 20$ are present.

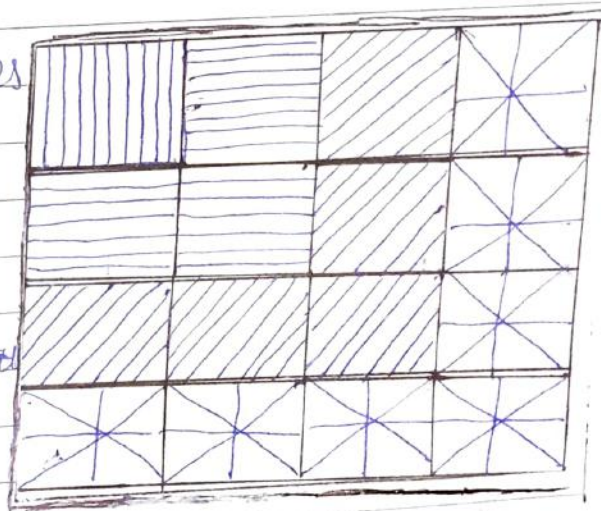
Task 3B :-

⇒ Here is one square.

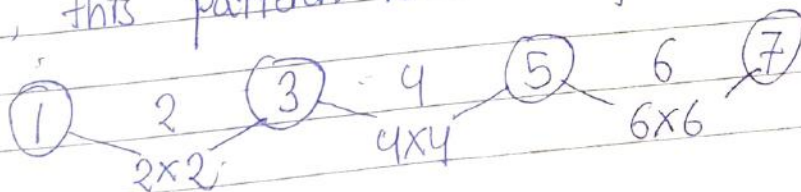
⇒ We can add 3 more squares to make 2×2 square.

⇒ Now we can ~~make~~ make 3×3 square by adding 5 squares more to 2×2 square.

⇒ Now we can make 4×4 square by adding 7 squares more.



* Yes, this pattern is related to the 1st pattern



Task - 302 :-Numbers

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

Square numbers

①
④
⑨
16
25
36
49
64
81
100
121
144
169
196
225
256
289
324
361
400

1, 4, 9, 6, 5,
6, 9, 4, 1, 0

1, 4, 9, 6, 5,
6, 9, 4, 1, 0

⇒ Yes, here we proved that the last digits
1, 4, 9, 6, 5, 6, 9, 4, 1, 0 repeats.

Task - 3C3 :-

I	II	III	IV	V	VI	VII	VIII	IX	X
(1)	2	3	(4)	5	6	7	8	(9)	10
11	12	13	14	15	(16)	17	18	19	20
21	22	23	24	(25)	26	27	28	29	30
31	32	33	34	35	(36)	37	38	39	40
41	42	43	44	45	46	47	48	(49)	50
51	52	53	54	55	56	57	58	59	60
61	62	63	(64)	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
(81)	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	(100)
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
(121)	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	(144)	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	(169)	170
171	172	173	174	175	176	177	178	179	180

If, a perfect square is divided by 10,
then the decimal is lying before 1, 4, 9, 6, 5, 6.
& the remainder should be zero.

Ex. $\rightarrow \frac{25}{10} = 2.5$ & remainder = 0.

Teacher's Signature & Date :

Task - 3c4 :-

* If the numbers are written in a 3-column grid then the all square numbers are lying on 1st column & 3rd column only.

* It doesn't lie on II column.

I	II	III		I	II	III
(1)	-	(9)		(64)	-	(81)
(4)	-	(36)		(100)	-	
(16)	-					
(25)	-					
(49)	-					

I	II	III		I	II	III
(121)	-	(144)		(169)	-	(225)
(169)	-					

Task - 3C5 :-

I	II	III	IV		I	II	III	IV
(1)	2	3	(4)		93	94	95	96
5	6	7	8		97	98	99	(100)
(9)	10	11	12		101	102	103	104
13	14	15	(16)		105	106	107	108
17	18	19	20		109	110	111	112
21	22	23	24		113	114	115	116
(25)	26	27	28		117	118	119	120
29	30	31	32		(121)	122	123	124
33	34	35	(36)		125	126	127	128
37	38	39	40		129	130	131	132
41	42	43	44		133	134	135	136
45	46	47	48		137	138	139	140
(49)	50	51	52		141	142	143	(144)
53	54	55	56		145	146	147	148
57	58	59	60		149	150	151	152
61	62	63	(64)		153	154	155	156
65	66	67	68		157	158	159	160
69	70	71	72		161	162	163	164
73	74	75	76		165	166	167	168
77	78	79	80		(169)	170	171	172
(81)	82	83	84		173	174	175	176
85	86	87	88		177	178	179	180
89	90	91	92		181	182	183	184

* The square numbers are lying on the 1st and 4th column.

Task - 3C6 :-

Yes, I can make such grids with other number of columns. Yes, I can predict where the square numbers would be with such grids.

Task - 4 :-

(i) (a) 122, (b) 356, (c) 340, (d) 12356 & (e) 425 are not perfect square numbers because,

(a)	(b)	(c)	(d)	(e)
$2 \overline{)122}$	$2 \overline{)356}$	$2 \overline{)340}$	$2 \overline{)12356}$	$5 \overline{)425}$
61	178	170	6178	85
	89	85	3089	17
		17		

By Prime factorisation method it can't be pairs. Therefore, they are not perfect square numbers.

(ii) (a) An example which is not a perfect square & whose unit digit is 0, leaves remainder 4 when divided by 8 is 20.

(b) Whose unit digit is 6, leaves a remainder 4 when divided by 8 and leaves a remainder 1 when divided by 3 is 66.

Jawahar Navodaya Vidyalaya
SALBANI, MAYURBHANTJ, ODISHA

Exploring Patterns in Square Numbers

Name:- Bhakani Sankar Sarangi

Class:- IX

Section:- A

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