LU 8.11. An experiment on measuring volumes

Overview

The story of crow and the pitcher (a kind of pot for holding water) from the collection of Aesop's Fables is very popular among children. Using this simple story, we can explore the concept of volume of solids and liquids with the students. In this Learning Unit, students learn how to estimate the volumes of different bodies by immersing them in water. We also introduce the idea of the packing of solids, i.e., when you try to pack together (or put together) many solid objects, there may be some gaps in between. Due to this property of solids, the crow may not be successful in raising the level of water beyond a certain limit.

Minimum time required: Three sessions of 40 mins each.

Type of Learning Unit: Classroom

Unit-specific objectives

- To make a marked transparent cylinder and use it to measure the volume of liquids.
- To use the volume of fluid displaced by submerged bodies to measure the volume of solids that do not dissolve in water.
- To understand the importance of the least count of a measuring instrument, and its use in finding the accuracy of volume measurement.
- To develop an intuitive understanding of the concept of packing fraction.

Links to curriculum

1. NCERT Class 7 Science Textbook: Chapter 9, Soil (relates to the concept of water that can be held in pore spaces of soil/rock pieces)
2. NCERT Class 8 Mathematics Textbook: Chapter 11, Mensuration Modified (the formulas from this chapter are not directly required here, but this unit helps students internalise some of the ideas of volume measurements)

Learning from "The crow and the pitcher" story

Do you remember the childhood fable of the crow and the pitcher? (See figure 1.) In this unit, we will imitate the crow in the story and use the concept that 'a body submerged in water displaces an amount of water equivalent to its volume', to carry out some measurements. The last task in this unit is closely related to the tale — and you may reach a surprising conclusion at the end of it!

Figure 1: *The Crow and the Pitcher*
From The Aesop for Children, by Aesop, illustrated by Milo Winter, Project Gutenberg etext 1994

Materials

- A narrow transparent cylinder (or a transparent 500 mL water bottle with the top cut off; the cylinder need not have uniform diameter across its length)
- Glass marbles (~40) of similar size
- Small irregular stone which can fit into the cylinder comfortably (see note in task 4)
- Ruler
- Marker pen (fine tipped)
- Straight edge (like another ruler or edge of a notebook)
- Beaker (with graduated volume markings)
- A tray or tough may be kept to collect water spills

Are you familiar with these ideas?

- Volume

Students should be familiar with the concept of volume in general and also the formula for volume of a sphere.

- Displacement of fluids by solid objects

Students should be aware that when a non-porous solid body is immersed in a fluid, it displaces fluid equal to its own volume. Familiarity with the Archimedes' principle of buoyancy is **not** necessary for this task.

- Average/ Mean

Students should be familiar with the concept of the average of several quantities.

Task 1: Creating your own volume measuring instrument (a graduated cylinder)

i. Use a beaker to carefully measure 50 mL water and transfer it to the transparent cylinder. Mark the height of the water column on the cylinder using a marker pen.
ii. Repeat this till the cylinder is almost full, marking successive heights at the steps of 50 mL.
iii. Label the markings with appropriate multiples of 50 mL. (50, 100, 150,...)

Now, you have a graduated cylinder which measures volume. You will notice that we can use this cylinder to measure volume only in multiples of 50 mL. Hence 50 mL is the least count of this graduated cylinder. If the water level is between two markings, we take the reading as the mark that is closest to the water level.

The maximum volume your graduated cylinder can measure is ________________________
(Highest marking on the cylinder)

- Here it would be useful to discuss the concept of least count with the students. In this context, the least count would be 50 mL.
- In task 4, we will see how to increase the accuracy of this graduated cylinder beyond the current least count.
- The graduated cylinders prepared by the students in this task can also be reused for other experimental purposes.
- You may make suitable modifications to the instructions above. For example, if there is a beaker which measures 25 mL instead of 50 mL, then the transparent cylinder could have markings for every 25 mL. However, reducing this least count of the cylinder should be avoided as it may increase error in measurements because of non-uniformity in shape of the

bottle and errors in making marks. Lower least count of this cylinder may also prompt students to take fewer number of marbles, which will lead to measuring volume change for lesser number of marbles increasing the relative error in volume per marble.

- Some labs may have a graduated beaker, or a graduated transparent cylinder, which is already marked. If the beaker or cylinder is large enough it could be used directly by one of the student groups. But do encourage the students to also prepare their own measuring cylinder. The results obtained with the marked cylinder and the beaker could then be compared. Before beginning, ask the students to observe the graduated beaker carefully and find its least count.

Task 2: Measuring the average volume of marbles

i. Take the empty graduated cylinder and fill it up to the 200 mL mark.

ii. Drop the marbles in the cylinder, one by one while counting them, until the water level rises up to the next mark. Ensure that **all** the marbles are **fully** submerged in water. That is, the level of the water should be above all the marbles. The water level rises because each marble displaces an amount of water equal to its own volume.

Volume of water before adding marbles ________________________________

Volume of water after adding marbles ________________________________

Number of marbles required to raise the water level to the next mark _________________
Thus, ______ marbles displace __________ volume of water.

iii. Use this result to estimate the average volume of one marble, obtained experimentally (V_{exp}).

Average volume of one marble _________________________

- It is possible that the students may not get the water level exactly up to a mark on the transparent cylinder. If the water level is below a certain mark, for say 20 marbles and goes above this mark for 21 marbles, then in such case, the number of marbles making up the difference in volume may be taken as 20 or 21 depending on if the 21st marble is submerged more or less than half and closeness of water level to the mark.
- Moreover, the topmost marbles must be submerged while measuring volume. If there are air spaces between the top marbles, some marbles may be removed till such spaces are no more observed. Altenatively, the task may be repeated with a larger starting volume of water.
- The students may be given a hint to first obtain the volume for "n" marbles in this task and then the average volume of one marble. Note that all the marbles are nearly equal in volume, but not exactly equal. This is why it makes sense to talk og an average volume and take it as an estimate of the volume of one marble.
- When we performed this task, the result we obtained for the set of marbles that we had was about 50 mL for 25 marbles, which meant that the average volume of one marble was about 2 mL.

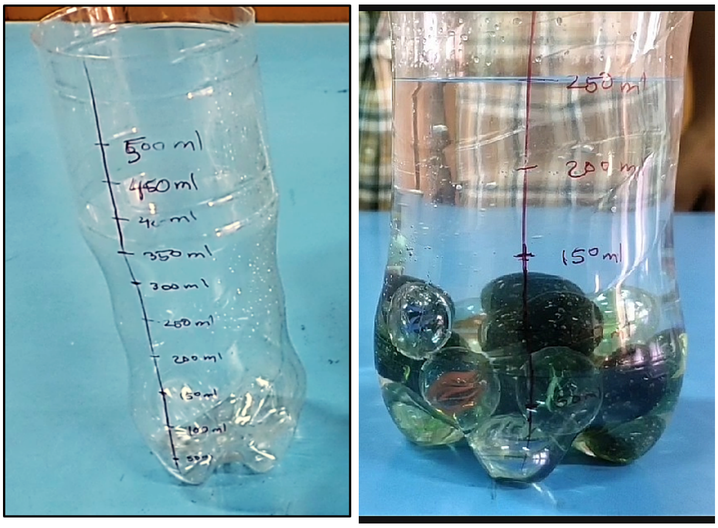

Figure T1: *(Left) A measuring cylinder and (right) A measuring cylinder with water and marbles*

Task 3: Comparing the volume of a marble estimated by two different methods

i. Keep ten marbles in a straight line touching each other. (You can create a long narrow channel by placing a straight edge and a ruler parallel to one another with a gap in between, with the marbles lined up in the gap.)

ii. Measure the end-to-end length of the line of marbles.
End-to-end length of ten marbles ______________________________

iii. Use this measurement to estimate the average radius of the marbles.
Average radius of one marble ______________________________

iv. Calculate the volume of a marble (sphere) $\left(V_{calc}=\frac{4}{3}\pi r^3\right)$ using the radius you have obtained.

Volume of one marble (obtained using the formula) ______________________________

v. You may notice that the volumes obtained by these two methods differ slightly from each other. One can estimate percentage difference as the ratio (expressed in percentage) of the difference in volume to the volume of a marble (by either method).
Percentage difference = ________________________________

- Dividing the end-to-end length by the number of marbles will give the mean diameter of the marbles. Have a discussion with the students on whether the diameter of each marble is exactly equal to the answer they wrote. If not, what does this answer signify?
- For our set of marbles, the end-to-end length of the line of marbles was (for 10 marbles) ~ 15.1 cm, i.e., the average diameter of a marble was about 1.51 cm. This gives the volume of one marble as $\frac{4}{3}\pi r^3=(\frac{4}{3})\times(\frac{22}{7})\times(\frac{1.5}{2})^3$ = 1.77 ml.
- The expression for the percentage difference can be written as: $\frac{|V_{calc}-V_{exp}|}{(V_{calc}\text{ OR }V_{\exp})}\times 100$ %
- We are using the percentage difference instead of percentage error, because among the two methods, we cannot say which is the accurate measurement. Both methods have inherent

possibility of measurement errors.

- The teacher can explain the concept of error qualitatively as follows: There is a clear discrepancy in the volume of one marble obtained in task 2 and that obtained in task 3. This is because in task 2, we calculate the average volume of one marble directly without finding the radius of marbles. On the other hand, in task 3, we find the average radius of a marble and then calculate the volume. In this process, any error in the measurement of the radius leads to larger error in volume as the radius gets cubed (due to $\frac{4}{3}\pi r^3$). This error can be minimised by increasing the number of marbles taken to measure the end-to-end length and then calculating the corresponding average radius of one marble.
- Note that the mean volume of marbles is obtained here to a greater accuracy than the accuracy imposed by the least count of our "manufactured" measuring cylinder.
- Possible extension: If the students want, they can repeat this with more marbles and/or with random sets of ten marbles each. Will the calculated average volume now be closer to that obtained in task 2?
- Possible extension: Students may also try repeating the tasks 2 and 3 using machined steel balls (like what is used in ball bearings) of a size similar to that of the marbles. Since these are usually manufactured to a higher degree of precision than the marbles, the discrepancy in the volume obtained in task 2 and that obtained in task 3 will be smaller.

Task 4: Measuring the volume of an irregular stone

i. Fill the cylinder with water to the 200 mL mark.

ii. Put an irregular stone in the water. (The stone should be completely immersed inside the water with the water level at least 2-3 cm above the upper surface of the stone.)

iii. Estimate the volume of the stone by observing the amount of water displaced. Unless the water level matches with one of the markings, this will only be approximate measurement.

iv. Now, immerse enough marbles to bring the water level up to the next marking.

Volume of water before adding the stone ______________________________

Number of marbles required to raise the water level to the next mark _____________________

The irregular stone + _________ marbles displaced __________ volume of water.

v. Use the mean volume of marbles, V_{exp} obtained in task 2 to determine the volume of the stone more precisely.

Volume of the irregular stone _______________________________

Note that once the stone is submerged, there needs to be sufficient depth of water above the stone to immerse a few marbles completely. If this is not the case the teacher may consider restarting with a larger amount of water.

Task 5: A challenge

In this task the students confront their belief that by putting enough number of marbles, the water level will always rise to the top of the container. They may be surprised that this is not the case. The main reason for this, which the students will discover through this task, is that there are gaps between the marbles, even when they are closely packed and the amount of water may be just sufficient to fill these gaps without rising above the level of the marbles.

1) Fill up the cylinder with water to the 50 mL mark.
2) By adding enough marbles, try to raise the **water level** to the top of the cylinder.
3) If you do not succeed in raising the water level to the top, can you estimate the maximum marking to which the water level rises?

   Maximum marking to which the water level rises ________________

   Number of marbles required to increase the volume by this amount ________________

4) Can you think of an explanation for this?

___

___

___

5) Do you think the thirsty crow would have succeeded in quenching its thirst? Explain your answer.

___

___

___

- The initial volume of water is kept deliberately low so that adding more and more marbles does not bring the water level up to the top. Once this happens, the teacher may support the students in finding the explanation that when objects like marbles are packed as closely as possible, they still have some gaps in between. The water accumulates there and hence never rises above a certain level. Assuming the closest possible packing of the marbles, there is still about a quarter of volume between the marbles that is left over for air or water. Thus, in this task, if the initial volume of water was 50 mL, the water level after adding many marbles will not rise beyond 200 mL. The teacher may also point out to the more interested students that this concept is called the “packing fraction”. Similar concept also becomes important in the theory which explains how atoms are packed together in solids.
- Thus, one of the takeaways of this unit is that the thirsty crow would not have succeeded in quenching its thirst if the water level in the pitcher had been too low to begin with.
- Only for the teachers: In case of (spherical) marbles, the closest possible packing is called “hexagonal close packed”. The packing fraction for this is $\frac{\pi}{3\sqrt{2}}$ = ~ 0.74. This means that in the most ideal closest packing, 74% of the volume will be occupied by marbles and at most 26% by water. Further, on the edges of the cylinder, this fraction will be poorer. Hence, we expect water in the task above to rise to between 120-150 mL.

Task 6: Estimating the packing fraction (Optional)

i. Fill the marked cylinder with marbles up to the 50 mL mark. Count the marbles as you are putting them in one by one. Give the cylinder a good shake to ensure that the marbles are packed as closely as possible and make sure that the marbles are as close as possible to the level with the 50 mL mark by adding or removing some marbles.

ii. The packing fraction is the ratio of the volume of the marbles filling up the space to the total volume of the space. Can you find the packing fraction using this definition?

iii. Repeat this for the 100 mL and the 150 mL mark and find the packing fractions for these trials as well. What do you notice?

- It should not be difficult for the students to find the average volume of one marble by calculation since they know the average radius of a marble. Multiplying by the number of marbles gives the total volume of the marbles. They already know the total space inside the container from the marked volume (50 mL, 100 mL, or 150 mL as the case may be). So they can find the ratio, which is the packing fraction.
- This is slightly more challenging: Another method is to plot a graph with the total volume of the marbles on the Y-axis and the total volume of the space occupied by them on the X-axis. By drawing a “best fit” straight line through the points, the slope of the line gives the packing fraction.
- An interesting extension of this activity is to compare the packing fractions obtained by using ball bearings and marbles together, and by varying the diameter of the container.
- It may be worth expanding the analogy to explain that groundwater is also filled in the pore spaces of soil particles and stones beneath the ground. Thus even though pore spaces in the soil seem tiny, they hold huge amounts of groundwater that we extract via hand pumps and borewells.

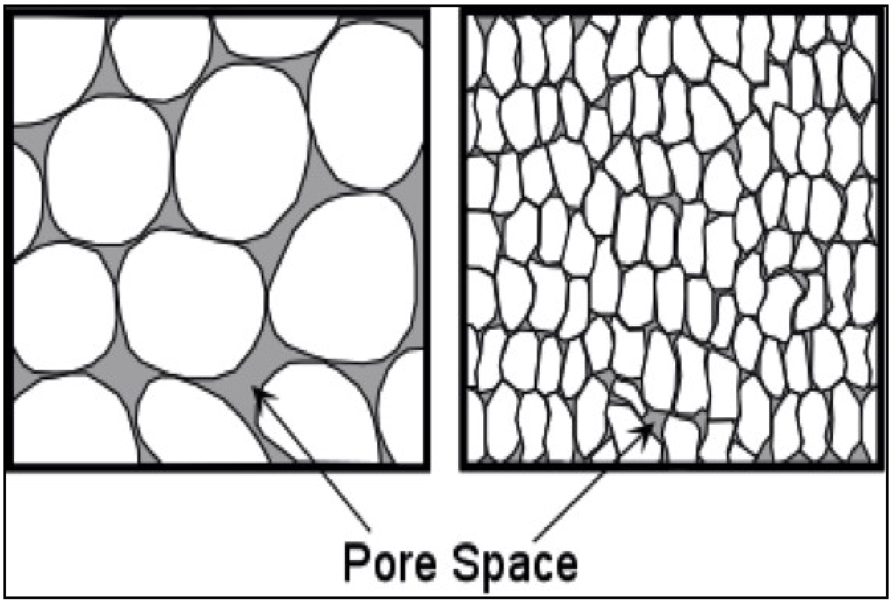

Figure T2 *Pore space between soil particles*

- Figure T3 below shows “hexagonal close packing” that you may have observed in the stacking of balls or of fruits in the market.

Figure T3 *Fruits stacked in a fruit stall*

Idea for further extension

What happens if we use a bottle with a larger or smaller diameter, of if we use marbles that are larger or smaller? Does the packing fraction change?

Suggested readings

1. An advanced discussion on pores in soils and their relationship to various processes and phenomena taking place in soil can be found at:
   Nimmo, J.R., 2004, Porosity and Pore Size Distribution, in Hillel, D., ed. Encyclopedia of Soils in the Environment: London, Elsevier, v. 3, p. 295-303. Retrived from:
   https://wwwrcamnl.wr.usgs.gov/uzf/abs_pubs/papers/nimmo.04.encyc.por.ese.pdf
2. The overflow principle, the principle that when a body is immersed in a liquid, it displaces an equal volume of the liquid, is often confused with the related but different Archimedes principle. The latter states the equivalance of the force of bouyancy and the weight of the displaced liquid. For an explanation see this link:
   https://www.math.nyu.edu/~crorres/Archimedes/Crown/CrownIntro.html

Credits
Main Authors: Ananda Dasgupta, Keyuri Raodeo
Contributing Author: Aniket Sule
Reviewers: Arnab Bhattacharya, Vandana Nanal
Editors: Beena Choksi, Geetanjali Date, Ankush Gupta, Reema Mani, K. Subramaniam