Where is My Shadow?

Introduction

Shadows are a phenomenon we experience every day; however, we may not have noticed how they change over a year. In this unit, we will try to understand how and why these changes occur. First, we will examine secondary data (graphs of shadow length) and try to understand patterns of change in shadow length over the year. Based on our understanding, we will make some predictions about our own shadow, which we will verify through observation and measurement. Finally, through an explanation of the apparent celestial motion of the sun, we will understand why shadows change over the year.

The unit is divided into these parts:

- **Story:** This is a story you can read this story before you start, or you can skip ahead to Session 1 on page 5
- **Student version:** Contains text, questions, and explanations for students. If you are doing this in class, your teachers will decide how to use this: they can read it aloud to the whole class, project it on screen, or provide printouts to students in groups or individually
- **Optional Questions:** These are questions to think about and for you to explore on your own, after class

Materials Required

1. Flat ground
2. Direct sunlight
3. Measuring tape / meter stick
4. Graphs and tables appropriate to your location

Note: By default, the data in this unit is provided assuming the location to be Pune. **To obtain data for your location**, contact vp@hbcse.tifr.res.in

Story

(You can read this story before you start, or you can skip ahead to Session 1 on page 5)

Mala's birthday is on 21 Dec. Mala has a birthday tradition: every year on her birthday and half birthday, she stands up, as straight as she can, with her feet fully on the ground, against a part of the kitchen wall and her father marks her height on it with a pencil. Mala and her father then use a measuring tape to measure the height and write it, in meters, next to the marking. She turned 13 in 2019 and to enter her teens, she wanted to add a new tradition.

The month before her birthday, Mala's class did an activity on measuring shadows. She wondered if her shadow grew with her. She decided to add this to her birthday tradition:

> Mala: "From now on, when we measure my height on my birthday and half birthday, can we also measure the length of my shadow? Exactly at 12pm."
>
> Father: "Oh, at 12pm won't your shadow just be zero?"
>
> Mala: "Well, last month our teacher had the whole class go out at 12pm and measure our shadows. I expected it to be zero, but it wasn't! Shorter students had shorter shadows, taller students had longer shadows."
>
> Father: "Yes, that makes sense. The shadow of the tree outside our house is always longer than my shadow. And I think, as you grow taller, your shadow will get longer too!"
>
> Mala: "But you know what's funny, after we measured our shadow, our teacher asked us all to divide the length of our shadows by our height, and we found that everyone ended up with the same number!"

Here is the data Mala and her friends recorded. Can you calculate $\frac{shadow\ length}{height}$ for each of them:

Date: 21 Nov 2019	**Shadow Length (m)**	**Height (m)**	$\frac{shadow\ length}{height}$
Mala	1.13	1.39	
Tara	1.17	1.45	
Zaraan	1.15	1.42	
Birju	1.12	1.38	

Data from Mala's class on 21 November 2019

> Father: "Hmm. So my shadow may be longer than yours because I am taller, but my $\frac{shadow\ length}{height}$ will be the same as yours? That's interesting."
>
> Mala: "Yes, and that's not even the most interesting part! We then measured the length of the shadow of a 1m stick and we found that this was the same as the number we had

all calculated!"

Father: "Aha! So this means that if you ever need to find the length of a shadow of a 1m stick, you can simply measure your own shadow and height and use that to calculate the shadow of a 1m stick at that exact time."

So Mala and her father measured her height, her fathers height, and both their shadow lengths at 12pm. They calculated each of their shadow lengths and divided it by their height.

They measured the shadow length of a 1m stick to be the same as shadow lengths and divided it by their height too, which confirmed what Mala had learnt in class.

Mala: "OK. I think we are convinced about our $\frac{shadow\ length}{height}$ being the same as the shadow length of a 1m stick."

Father: "OK, from now on, when we say shadow length, we just mean the shadow length of a 1m stick. Either through our calculation or the direct measurement of the shadow of a 1m stick."

Mala: "OK. I noticed that the shadow length has changed from when I measured it a month ago."

Father: "Do you think the shadow length will change every day? I want to see this for myself, let's try it."

Mala: "I guess it does change slightly in a month. But maybe it will be very different on my half birthday, 6 months from now. On 21 June, when we measure my height, we should remember to measure my shadow at 12pm and calculate the shadow of a 1m stick."

On her half-birthday, 21 June, Mala and her father measured their shadow lengths and heights again. They calculated $\frac{shadow\ length}{height}$, and even confirmed it by measuring the shadow of an actual 1m stick:

Date	Shadow Length (m)
21 Nov 2019	0.81
21 Dec 2019	0.92
21 Jun 2020	0.17

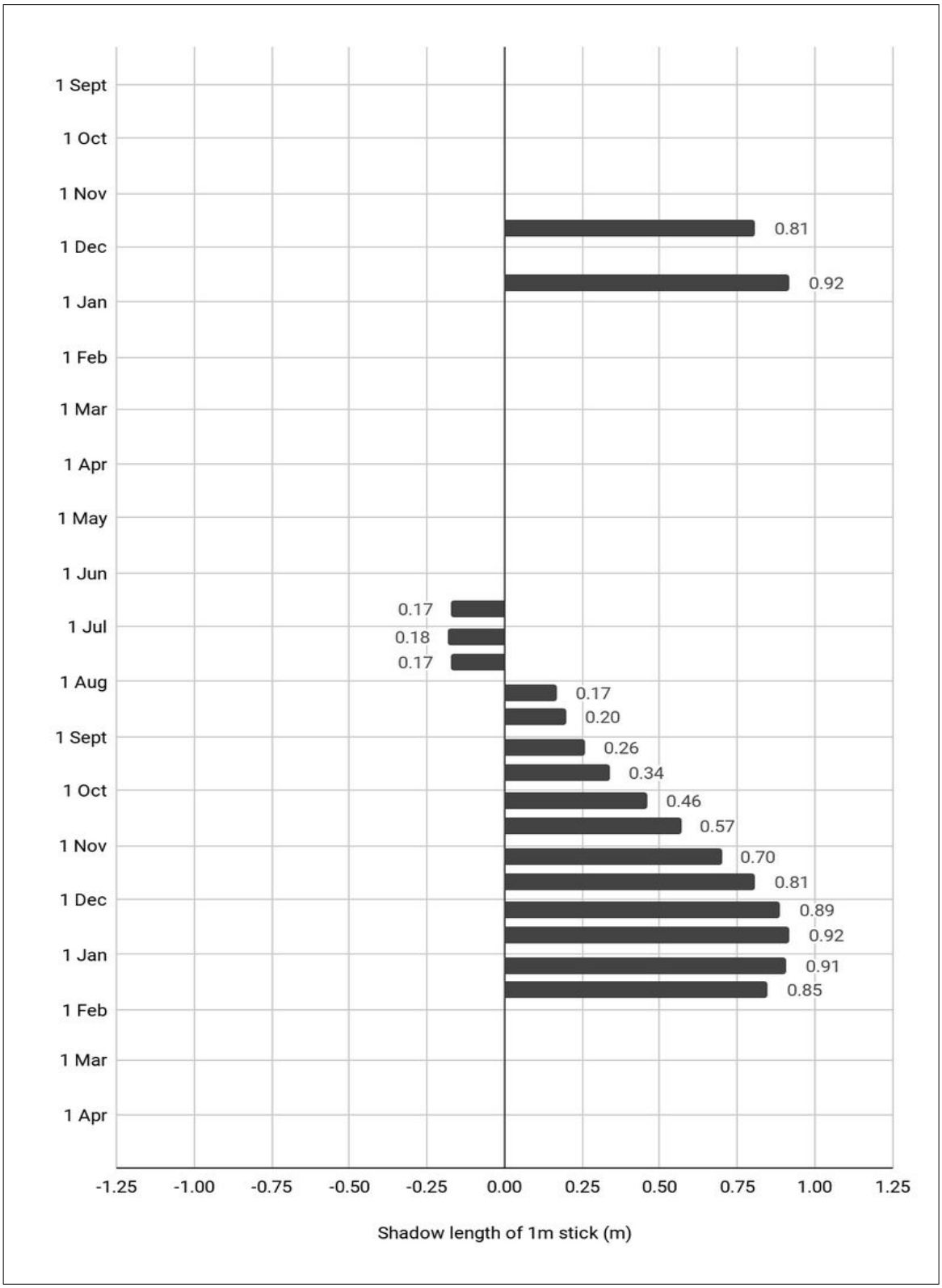
Shadow length of a 1m stick (calculated by Mala) in Nov, Dec, and Jun

Father: "Look, the shadow length has decreased since your birthday on 21 Dec!"

Mala: "Yes, it has. I want to keep measuring the shadow length regularly and see what happens to it."

So, Mala decided to measure her shadow every month. At 12pm, on the 7th and 21st of every month, Mala's father helped her measure her height and her shadow length. They kept doing this until her next

birthday. She then calculated the shadow of a 1m stick and plotted it on a graph.



Graph of shadow length of a 1m stick, plotted by Mala

By looking at this graph, Mala thinks she can guess what is happening to shadow length over the course of the year.

Session 1

Mala is a girl from your town. She did an activity where her class measured the length of their shadow at 12pm.

What do you think Mala's shadow length was at 12pm? _____

Task 1: Comparing shadow lengths

Here is the data Mala and her friends recorded at 12pm. Can you calculate $\frac{shadow\ length}{height}$ for each of them:

Date: 21 Nov 2019	Shadow Length (m)	Height (m)	$\frac{shadow\ length}{height}$
Mala	1.13	1.39	
Tara	1.17	1.45	
Zaraan	1.15	1.42	
Birju	1.12	1.38	

Table 1: Data from Mala's class on 21 November 2019

Mala and her friends noticed that while each student's shadow length may be different, the $\frac{shadow\ length}{height}$ will be the same. Their teacher told them that the number they calculate for $\frac{shadow\ length}{height}$ will be equal to the shadow length of a 1m stick.

Can you imagine why the shadow length of a 1m stick is the same as $\frac{shadow\ length}{height}$ of any object?

Task 2: Mala's measurements over a year

Do you think Mala's shadow length is going to change from one day to the next? One month to the next month?

Do you think her shadow length is going to change over the year? Why / Why not?

What do you think will happen to the length of the shadow of a 1m stick over the year?

Mala measured her height and shadow length, and used it to calculate the shadow of a 1m stick on her birthday (21 December) and her half birthday (21 June). She saw that the shadow length had changed. The table below shows her measurements.

Date	Shadow Length (m)
21 Nov 2019	0.81
21 Dec 2019	0.92
21 Jun 2020	0.17

Table 2: Shadow length of a 1m stick (calculated by Mala) in Nov, Dec, and Jun

After that she also measured her height and shadow length two times every month from July to December. Again, she used this to find the shadow length of a 1m stick. Then, she plotted all her data onto a graph, seen in Figure 1.

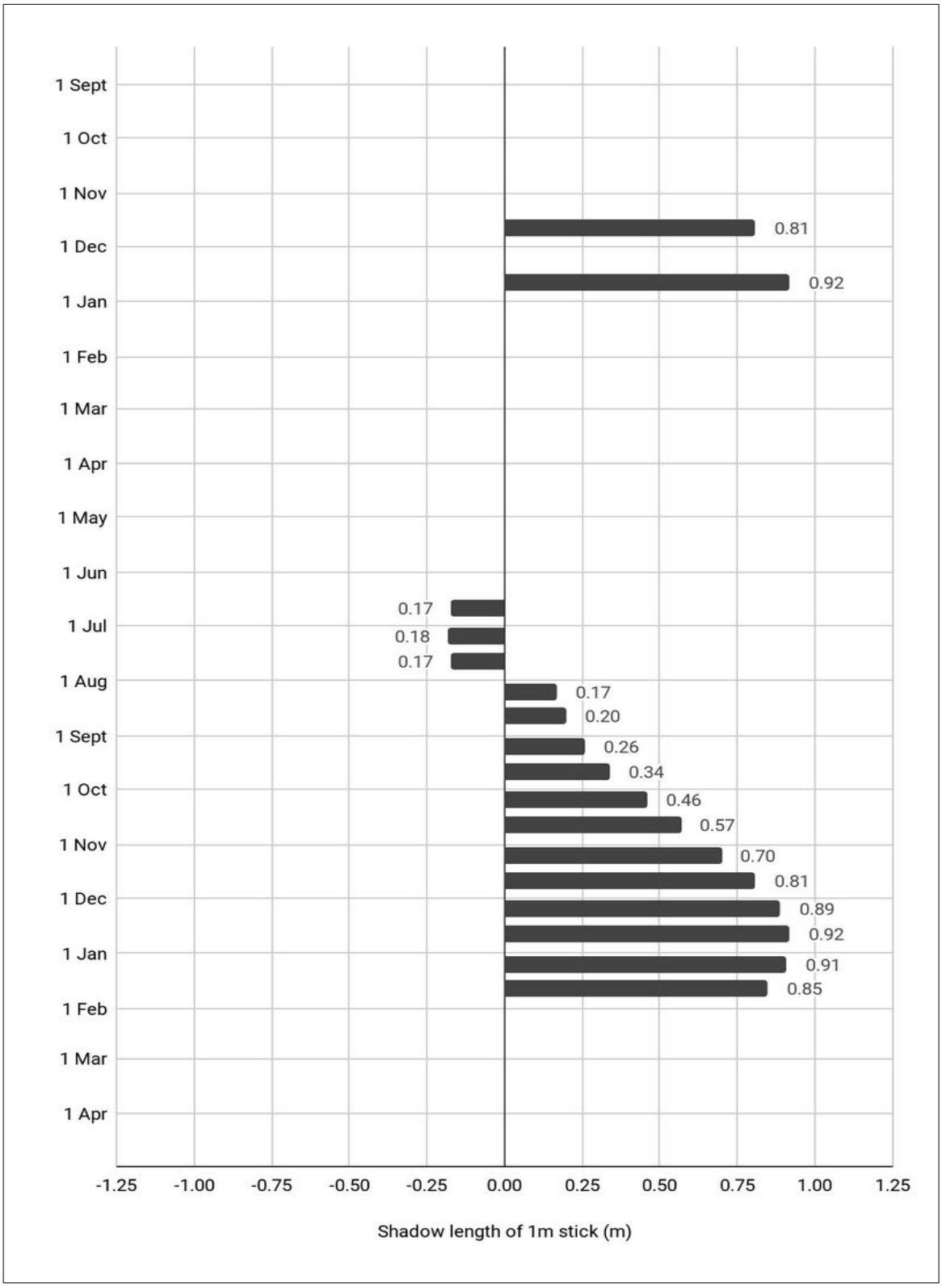


Figure 1: Graph of shadow length of a 1m stick, plotted by Mala

By looking at this graph, Mala thinks she can guess what is happening to shadow length over the course of the year.

Task 3: How does the shadow length change over the year?

Can you also figure out what is happening to shadows over the course of the year?

These questions about the graph may help:

What happens to shadow lengths on the same day of different years?

5. What is the shadow length of a 1m stick on:
   a. Mala's 13th birthday (21 Dec 2019) and her 14th birthday (21 Dec 2020)? Are they the same?
   b. One month before Mala's 13th birthday (21 Nov 2019), and one month before her 14th birthday (21 Nov 2020)? Are they the same?
6. Can you find out what the shadow length of a 1m stick was on:
   a. Two months before Mala's 13th birthday (21 October 2019) — mark it on the graph
   b. Three months before Mala's 13th birthday (21 September 2019) — mark it on the graph
7. Can you find out what the shadow length of a 1m stick is going to be on Mala's 14.5th half birthday (21 June 2021).
8. What can you conclude about shadow lengths on the SAME day of ANY year?

The shadow of a 1m stick on the SAME day of ANY year will be the same. So, you can think of data in the above graph as having been collected for 2021.

How does the direction of shadow (north / south) change?

9. between 21 September and 21 December, what do you think is happening to the:
   a. Length of the shadow?
   b. Direction of the shadow?
10. between 21 June and 21 September, what do you think is happening to the:
   a. Length of the shadow?
   b. Direction of the shadow?

When is the shadow maximum and minimum? Is the shadow ever zero?

11. What was the longest shadow length that Mala measured?
   a. Is there any point in the year where the shadow length at 12pm could be longer than what has already been measured? (Think about the months where shadow lengths were not measured.)
12. What was the shortest shadow length that Mala measured?
   a. Is there any point in the year when the shadow length could be zero?
   b. If you think the shadow is going to become zero, which day(s) do you expect it to be zero?

Task 4: What do you think is going to happen to your shadow over the year?

Now that you have made some sensse of the graph, try to make predictions about your shadow length. Fill these out in Table 3

1. What do you think happens to the length and direction of the shadow between January and June?
   a. On 21 February: More or less than 21 January, which direction?
   b. On 21 March: More or less than 21 February, which direction?
   c. On 21 May: More or less than 21 June, which direction?

   You can draw bars (in pencil) on the graph to show what you think is going to happen.
2. What do you think is going to happen to the shadow over the next two weeks?
   a. Will it increase or decrease (or something else)?
   b. Is it going to be to your south or north (or change direction)?

	Prediction (Task 4)		**Actual (Task 6)**	
Date / Time period	**Length**	**Direction**	**Length**	**Direction**
Shadow length on 21 Jan				
Shadow length on 21 Feb				
Shadow length on 21 May				
Your shadow in the next two weeks				

Table 3: Make your predictions here, and check back with your actual data

Task 5: Measure your shadows and check your predictions

Measure your shadow every day:

- Find a spot where there is direct sunlight at 12pm. Make sure the ground is more or less flat. Avoid ramps / inclines.
  - Make sure your clocks are correct! Measurements need to be done at exactly 12pm IST.
- Observe and measure your shadow:
  - Make a note of the direction: is your shadow towards the north or south?
  - Measure shadow length: start from the point between your feet and measure till the shadow of your head.

Take a measurement every day for the next 2-3 weeks and record it in Table 4.

Finally, plot the data from Table 3 onto Figure 1.

Session 2

Task 6: How did the shadow length change during the past weeks?

Fill these out in Table 3 (see page 9)

1. In the past two weeks, did your shadow length
   a. increase
   b. decrease
   c. stay the same
   d. [other]

   Was this what you predicted?
2. In the past two weeks, was your shadow direction
   a. To the north
   b. To the south
   c. Change from north to south
   d. Change from south to north

   Was this what you predicted?
3. Does your data help you to predict what is happening to the shadow between 21 January and 21 June?
4. Why do you think shadow length changes over the year?

To understand why shadows change, let's look more closely at how the sun moves in the sky over the year.

Task 7: How does the angle of the sun change over the year?

Using the shadow length and the stick, we can draw how the rays of the sun fall on the stick to make the shadow. Since the sun is very large and very far away from the earth, we can assume that all the sun's rays reaching the earth are parallel. You will notice that the sun's rays seem to fall at an angle, even though the shadow is formed at 12pm.

Below is a diagram to see the shadow formed by a 1m stick on 21 December. In the diagram, you can see a grid. One box in the grid represents 10cm or 0.1m. The 1m stick is 10 boxes high. For example, if the shadow formed on 21 December is 0.92 m, the length of the shadow is 9.2 boxes long. And if we know that the shadow falls to the north, it is drawn in that direction.

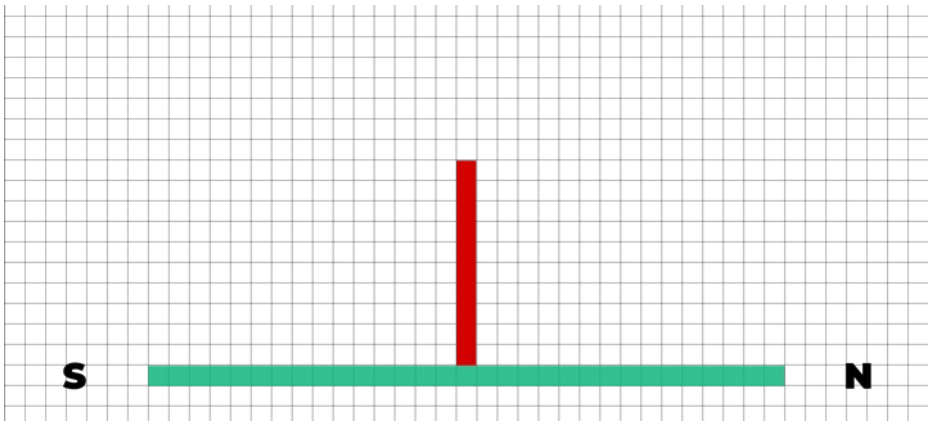


Fig. 2a: Shadow length diagram

We will use diagrams like the one above to do some tasks.

Using Figure 2a and the instructions below, find the angle of the rays for 21 December.

- Draw the shadow formed on 21 December. (Remember, one box in the grid represents 10cm or 0.1m.)
- Draw the sun's rays that cause this shadow to form. Make sure the rays you draw are all parallel to each other
- Measure the angle of the sun's rays using your protractor. Is the angle the same or different from the angle you measured for 21 December?

You can use the same method to find the angle made by the sun's rays on other days of the year.

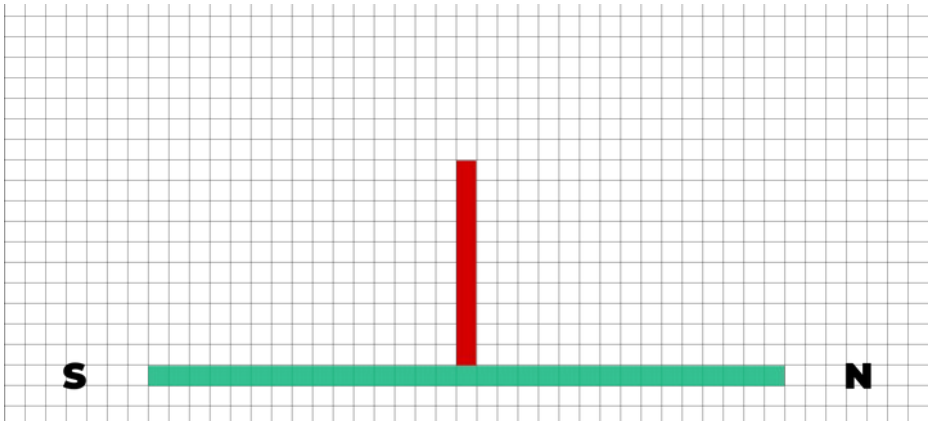


Fig. 2b: Blank ground and stick (for drawing)

Use Figure 2b to draw the shadow and the sun's rays, and measure the angle of the sun's rays, for 21 June.

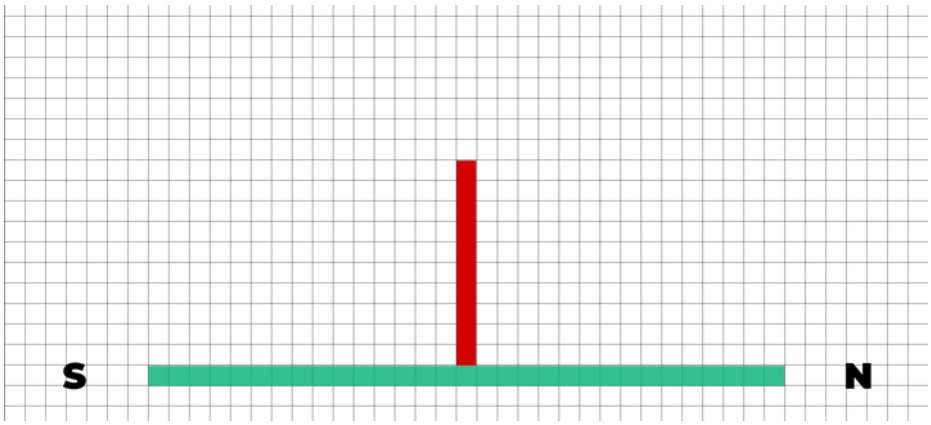
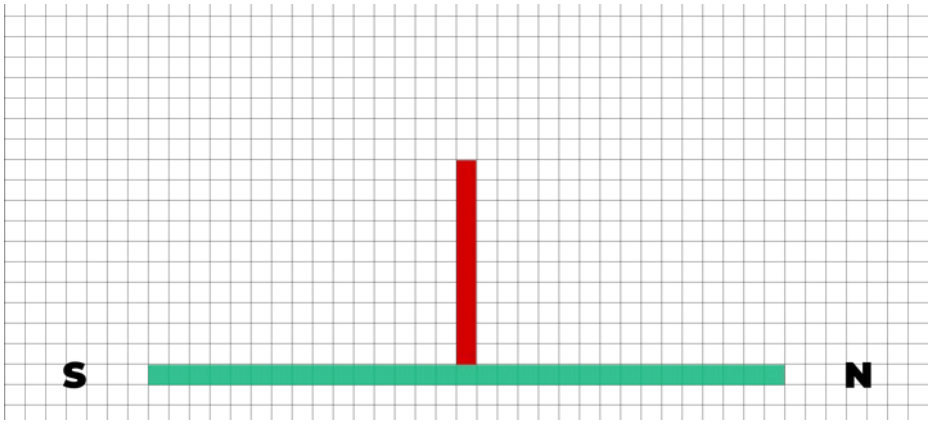


Fig. 2c: Blank ground and stick (for drawing)

Use Figure 2c to draw the shadow and the sun's rays, and measure the angle of the sun's rays, for 21 September

Fig. 2d: Blank ground and stick (for drawing)



Finally, choose any other day from Figure 1. Use Figure 2d to draw the shadow and sun's rays, and measure the angle of the sun's rays, on that day.

Look at the four diagrams. What happens to the angle of the sun's rays over the course of the year?

Task 8: Why does the angle of the sun's rays change over the year?

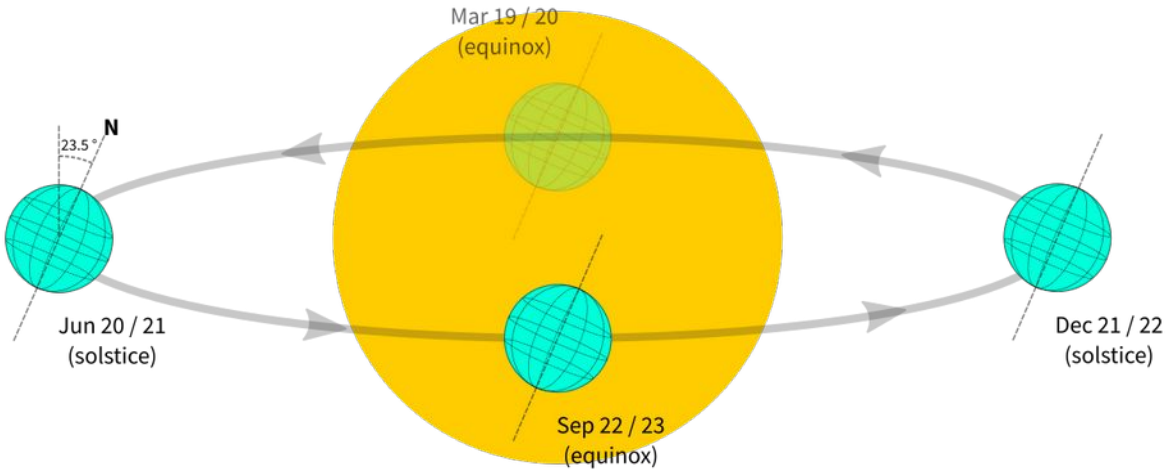
The angle of the sun's rays at 12pm depends on 2 different things:

1. Where the earth is in its revolution around the sun
2. How far north (or south) you are on the earth, i.e., your latitude. For this explanation, we imagine that Mala is in Pune, which is at 18° N.

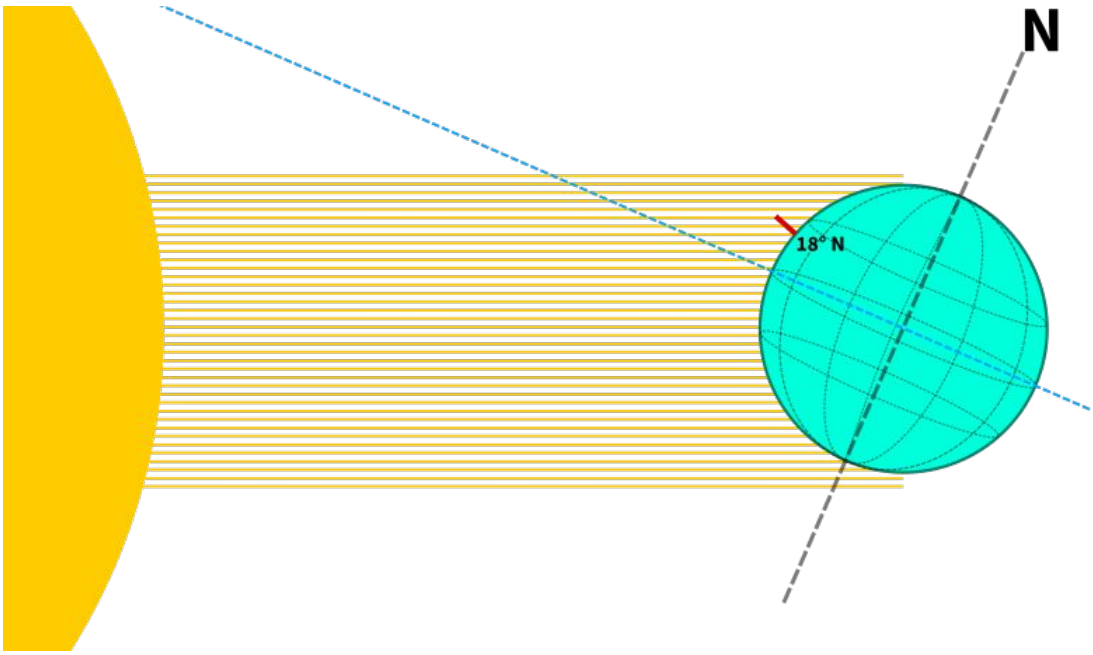
We will try to understand how (1) makes the angle of the sun change.

Note for all figures in this section: Sizes are not to scale. The Sun is not in the exact centre of the orbit. The eccentricity shown here is exaggerated.

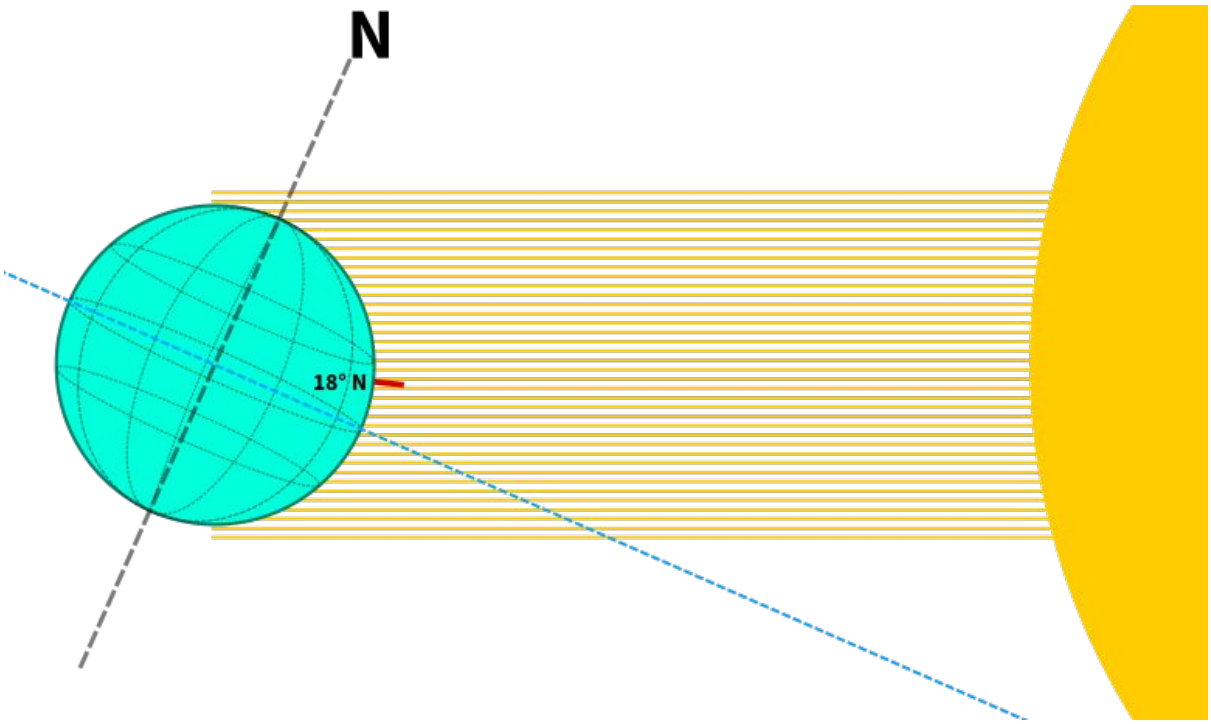
In the figure you can see the path of the revolution of the earth around the sun. Remember that the earth rotates about an axis that is tilted by 23.5°, as shown in the figure.



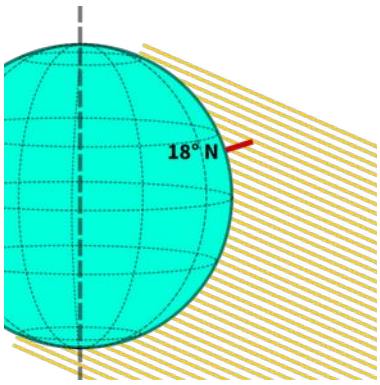
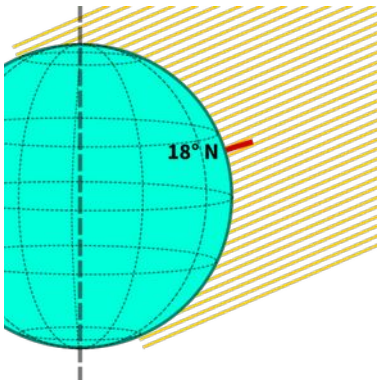
Imagine a stick placed in the ground in Pune, 18° N of the equator. The dashed blue line shows you where the equator is. Look at the position of the earth on 21 December, Mala's birthday. Because of the tilt of the earth's axis, the northern hemisphere faces away from the sun, and the southern hemisphere faces the sun. This is also the date of the December solstice. In this view, the sun appears to be south of the equator.

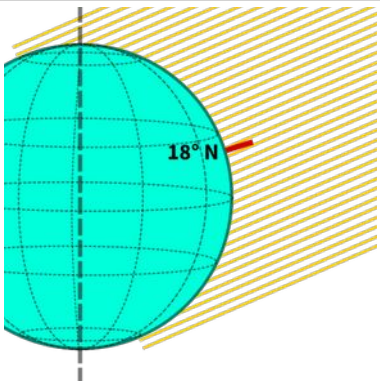
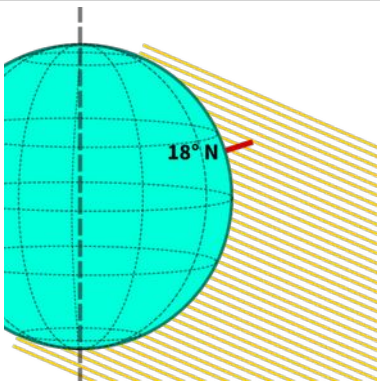


Now look at the position of the earth 21 June, Mala's half-birthday. Because of the tilt of the earth's axis, the northern hemisphere faces the sun, and the southern hemisphere faces away from the sun. This is the date of the June solstice. In this view, the sun appears to be north of the equator.

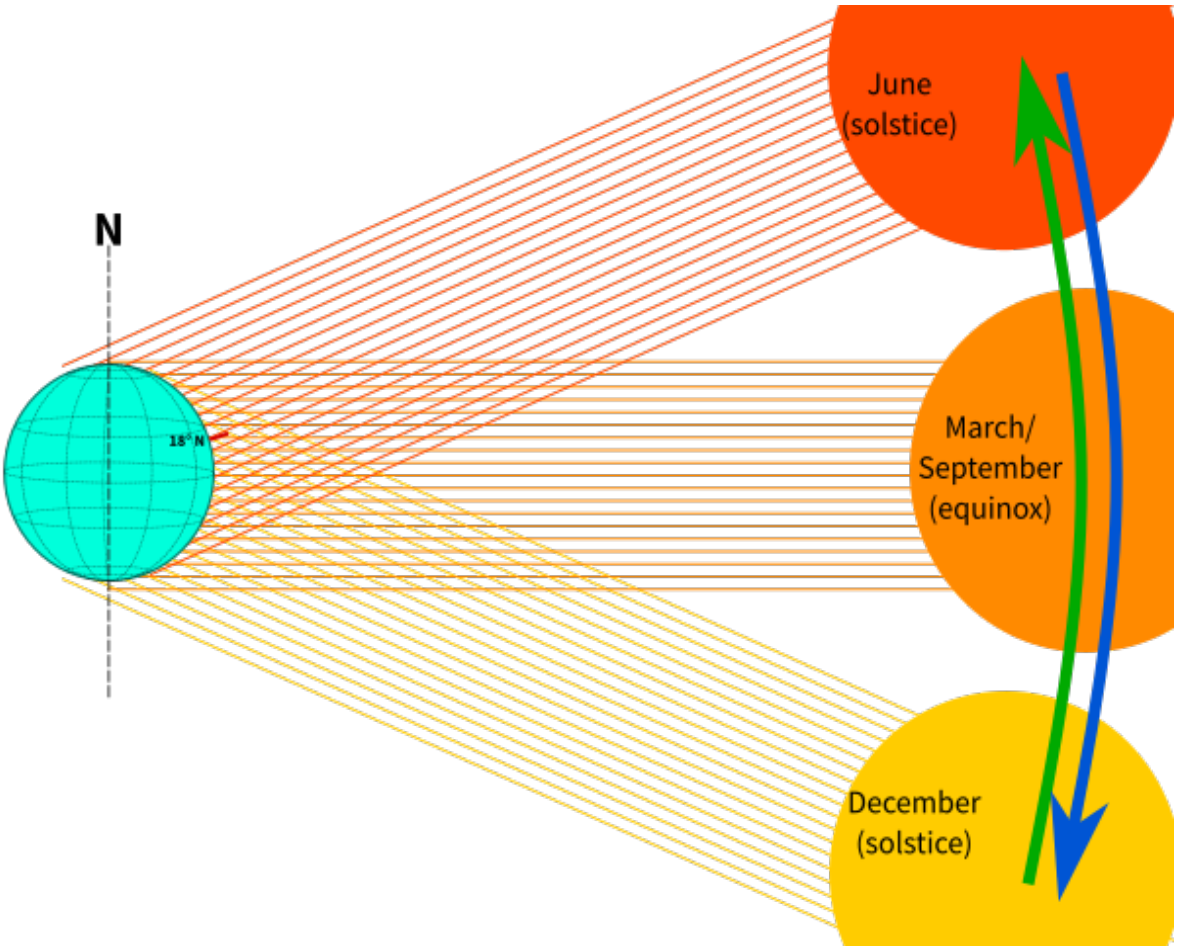


If the sun appears to be south of the equator in December and north of the equator in June, what happens in between?

		
December	March	June

		
June	September	December

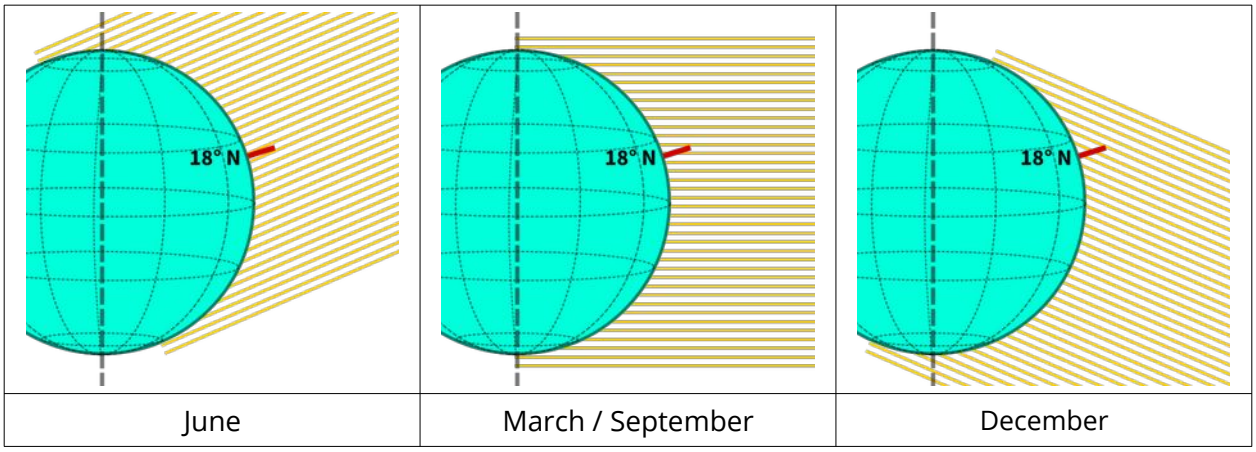
We can understand this better if we try to imagine where the sun appears to be at different times of the year. The figure below helps us see what happens.



So, from the December solstice to the June solstice, the sun will appear to move from south of the equator to north of the equator (green arrow). On the March equinox, the sun appears to be exactly between its December and June positions.

From the June solstice to the December solstice, the sun appears to move back from north of the equator to south of the equator (blue arrow). Again, on the September equinox the sun appears to be exactly between its June and December positions - exactly where it was at the March equinox.

Let's come back to our 1m stick in Pune at 18° N. From June to September to December, the angle the sun's rays are making with the stick changes gradually. If you look closely at this image, you can see how this angle is changing.



June	March / September	December

Task 9: Putting it all together

Based on what you understand about the movement of the sun, the angle of its rays, and the direction and length of the shadow, can you fill in Table 5 in your worksheet?

Time of the year	Hemisphere facing the sun	Apparent motion of the sun	Direction of the shadow	Angle of the sun's rays	Shadow length
December to March					
March to June					
June to September					
September to December					

Using this table, can you explain the measurements that Mala obtained?

It may help to see how the sun's rays change at your location. You can mark a stick at your latitude on Figures 3a — 3c. Do the angle of the sun's rays (approximately) agree with what you measured in Figures 2a — 2c?

Further Reading and Resources

'Zero Shadow Day' app

An Android smartphone app that contains a number of interactive visualisations to understand how shadows cast by the sun change over the course of a year at different places. Also provides data for users to examine. The app was commissioned by the Astronomical Society of India - Public Outreach and Education Committee (ASI-POEC). https://play.google.com/store/apps/details?id=com.alokm.zsd

Video about zero shadow day

A video by Arvind Gupta, explaining the phenomenon of Zero Shadow Day — why, how, and when does this day occur? https://www.youtube.com/watch?v=arciOj_70NQ

Poster about zero shadow day

A poster prepared by the Institute of Mathematical Sciences (IMSc), Chennai, about the arc made by the sun in the sky, how it changes over the course of a year, and how it causes zero shadow day. https://www.imsc.res.in/outreach/resources/ZSD_poster.pdf

ASI-POEC activities about shadows

Activities designed by ASI-POEC to help you explore and understand shadows

https://astron-soc.in/outreach/activities/zero-shadow-day/

https://astron-soc.in/outreach/activities/shadows-equinox/

A book of activities to explore the sun and shadows

Monteiro, V., Mahashabde, G., & Barbhai, P. (2008). Sun Earth Experiments: Activity Cards for Day-time Astronomy. Navnirmiti Learning Foundation. http://archive.org/details/SunEarthGames-DayTimeAstronomy

Questions to think about

How does the angle of the sun's rays change depending on how far north or south you are?

We said earlier that the angle of the sun's rays also depends on how far north (or south) you are on the earth, i.e., your latitude. Can you try to imagine this? If you would like to learn more, you can read Unit 2 of this set.

Can you imagine the time(s) of the year when your shadow has length zero?

We may think that at 12pm every day, our shadow becomes zero. However, there are only specific days in the year when this happens, and that too only in specific locations. On these days, the sun's rays have an angle 0° with the stick. So, the stick (and you) will have shadow length zero! These days are called Zero Shadow Days. In the tropics (between 23.5° N and 23.5° S), this happens twice a year. At the Tropic of Cancer (23.5° N) and Capricorn (23.5° S) this happens exactly once, on the June and December solstice respectively. Everywhere else on the earth it never occurs!

If you are in the tropics (between 23.5° N and 23.5° S), you can observe this phenomenon, twice a year. To check the exact dates and times that this happens in your location, download the ZSD app from ASI-POEC (https://play.google.com/store/apps/details?id=com.alokm.zsd&hl=en&gl=US).

Were the predictions you made correct?

You made some predictions about shadow length in Table 3. To test these predictions, you can measure the shadow on those days.

You can also figure out whether your predictions were correct by using the ZSD app from ASI-POEC. Play around with it and find out what the shadow length would be on these days.

(https://play.google.com/store/apps/details?id=com.alokm.zsd&hl=en&gl=US)