# Is Your Polygon Same as Mine?

## Introduction

Have you ever wondered how would you describe a triangle that is in your mind to somebody on a phone? What do you really tell, do you tell sides or angles? And is it possible, that the person would get the exact same figure that you had in mind? Moreover, how can you do that by giving minimum information? Today we will try to answer these questions by investigating some examples, making observations, and verifying or refuting these observations.

## Materials required

Blank sheets, Pencils, Erasers, Compass-boxes (compass, set-squares, protractor, and scale), scissors

## Task 1: Drawing Your Triangle

1. Draw a triangle of your choice on the given blank sheet of paper. Measure the sides and the angles of the triangle and name the triangle.
2. Now see triangles drawn by your friends. Do you see anything interesting? What is it?

_____

Keep the paper safe, on which you drew your triangle, we will be coming back to this triangle later in the activity.

## Task 2a: Constructing a Triangle When One Side is Given

You know that only one side of the triangle is 6 cm. Now using this information draw a triangle on the given paper. Name your triangle.

Now study the triangle drawn by your partner.

1. Is your triangle same as your partner's?

_____

2. How did you compare these two triangles?

_____

3. You and your partner both were given one side, 6 cm. Did you both get exactly the same triangles? Why?

_____

## Task 2b: Constructing a Triangle When One Angle is Given

You know that only one angle of the triangle is $55^{o}$. Now using this information draw a triangle on the given paper. Name your triangle.

Now study the triangle drawn by your partner.

1. Is your triangle same as your partner's?

_____

2. How did you compare these two triangles?

_____

3. You and your partner both were given one angle of $55^{o}$. Did you both get exactly the same triangles? Why?

_____

### Task 3: Construct a Triangle When Two Measures are Given

Group A: Draw a triangle whose sides are 5 cm and 3 cm. Name your triangle.
Group B: Draw a triangle whose one side is 6 cm and one angle is $55^0$. Name your triangle.
Group C: Draw a triangle whose two angles are $50^0$ and $75^0$. Name your triangle.
Now study the triangle drawn by your partner.

1. Is your triangle same as your partner's?

_____

2. How did you compare these two triangles?

_____

_____

Group A:
You and your partner, both were given two sides, 5 cm, and 3 cm.
3. Did you both get exactly the same triangles? Why?

_____

_____

Group B:
You and your partner, both were given one side, 6 cm and one angle $55^0$.
4. Did you both get exactly the same triangles? Why?

_____

_____

Group C:
You and your partner, both were given two angles, $50^0$ and $75^0$.
5. Did you both get exactly the same triangles? Why?

_____

_____

### Task 4: Construct a Triangle When Three Measures are Given

Group A1: Draw a triangle XYZ such that XY = 4 cm, YZ = 6 cm and XZ = 7 cm
Group A2: Draw a triangle ABC such that, AB = 5 cm, BC = 6 cm and $\angle ACB = 45^0$.
Group B1: Draw a triangle IJK such that $\angle IJK = 40^o$, $\angle JKI = 65^o$ and $\angle IKJ = 75^o$.
Group B2: Draw a triangle STU such that $\angle UST = 50^o$, ST = 3 cm and $\angle STU = 65^o$.
Group C1: Draw a triangle EFG such that EF = 7 cm, FG = 9 cm and $\angle GEF = 90^o$
Group C2: Draw a triangle PQR such that PQ = 5 cm, $\angle PQR = 50^o$ and QR = 4 cm

Now study the triangle drawn by your partner.
1. Is your triangle same as your partner's?

_____

2. How did you compare these two triangles?

_____

_____

### Task 5: Minimum Conditions For the Construction of a Unique Triangle

1. If you want your friend/partner to construct exactly the same triangle like the one you drew in Task 1, what minimum information you will have to provide such that she/he will also construct the exact same triangle?

_____

_____

2. If you think there are more than one ways to provide information such that a triangle, same as the one you drew in Task 1 could be constructed, please mention all those here.

_____

_____

### Task 6: Constructing a Quadrilateral

1. Now that you all know how to make a congruent triangle, let us figure out how to make a congruent quadrilateral. So if the minimum conditions for making a congruent triangle are three, what should be enough for a quadrilateral?

_____

Now given that all the sides of a quadrilateral are 3 cm, think about all the different quadrilaterals that you can draw. Make the constructions on the given blank sheet.

2. Did you or your partner get different quadrilaterals?

_____

3. So if only sides are given, is it always possible to get different quadrilaterals? How do you know?

_____

4. Imagine that you have to write to your friend about a quadrilateral. Now think of minimum information that you can send him/her, such that he/she gets the exact same quadrilateral as the one you had in your mind. What information you will send?

_____

_____

5. Think about why that set of information will lead to congruent or non-congruent quadrilaterals.

_____

_____

6. List the conditions that worked for constructing a unique quadrilateral.

_____

_____

### Task 7: Some Special Triangles and Quadrilaterals

We have found out the minimum information needed to draw congruent triangles and congruent quadrilaterals but let us look at some special triangles and quadrilaterals and find out what kind of minimum information we need to construct them.

1. How many conditions do you need to construct congruent equilateral triangles?

_____

2. How many pieces of information do you need to construct congruent squares?

3. How many pieces of information do you need to construct congruent rectangles?

4. How many pieces of information do you need to construct congruent rhombuses?

5. How many pieces of information do you need to construct congruent parallelograms?

6. How many pieces of information do you need to construct congruent trapeziums?

### Task 8: Constructing a Pentagon

1. Now that you all know how to make a congruent triangle or a congruent quadrilateral, let us figure out how to make a congruent pentagon. So if the minimum conditions for making a congruent triangle are three, and that for a quadrilateral are 5, what do you think the number of minimum conditions needed to construct a unique pentagon is?

2. Imagine that you have to write to your friend about a pentagon. Now think of minimum information that you can send him/her, such that he/she gets the exact same pentagon as the one you had in your mind. What information you will send?

Check whether what you suggested as minimum information really works. Try drawing different pentagons for the information you said you would give to your friend in the question above.

3. Think about why that set of information will lead to congruent or non-congruent pentagons.

4. List the conditions that worked for making a unique pentagon.

### Task 9: Finding the Number of Conditions to Construct Congruent Polygon

Now that you know the minimum conditions needed for making a congruent triangle and a congruent quadrilateral, let us explore how many conditions are needed for constructing a congruent hexagon, or a congruent heptagon.

Make some guesses, and make constructions on the given sheets of paper, record your guesses in the given table.

Table 1

| Number of sides in the polygon | Name of the polygon | Minimum conditions required for constructing a congruent polygon |
|---|---|---|
| 3 | Triangle | 3 |
| 4 | Quadrilateral | 5 |
| 5 | Pentagon | _____ |
| 6 | Hexagon | _____ |
| 7 | Heptagon | _____ |
| 8 | Octagon | _____ |

**Proving Our Understanding**

Let us find out how can we prove which guesses are right and which ones are wrong.
Draw a quadrilateral.
Draw the first triangle inside the quadrilateral.
See *Figure 3* here.
(Here we have drawn two different types of quadrilaterals)

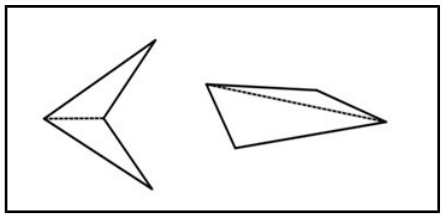

*Figure 1: Trigulating Quadrilaterals*

1. How many such triangles can be made inside each of these quadrilaterals?

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Yes, 2 such triangles can be made. We know that for constructing a congruent triangle we need 3 minimum conditions.
So in this case, to construct the first triangle we needed three minimum conditions. For the next triangle, we need three more, but as one side overlaps, you need only two conditions to construct triangle congruent to the second triangle. Also we know that to fix one point we need at least two conditions. Hence these 5 conditions are the minimum pieces of information needed to construct a quadrilateral.

This also reconfirms with our understanding of minimum conditions needed for constructing a congruent quadrilateral.
What will happen if we do the same for a pentagon?

Let us draw a pentagon and see how many triangles can be made inside a pentagon.

For the first triangle we need three conditions, for the second triangle we will need another 3 but then one side overlaps so we need only two. Similarly, for the third triangle, we need two more conditions.

So you can see that whenever you add a triangle, you add two conditions. So the minimum conditions necessary for constructing a congruent pentagon, are 7 (3 + 2 + 2). But these five conditions give us a unique pentagon, hence the minimum pieces of information needed to construct a unique pentagon is 7.

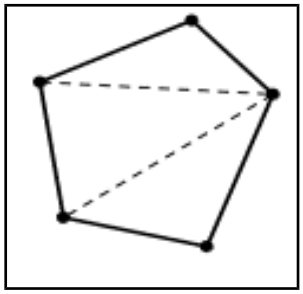

*Figure 2*

Let us try to figure this out for hexagons, heptagons, and octagons.

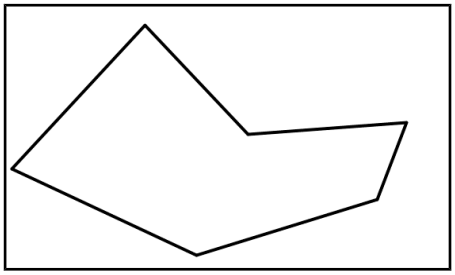

*Figure 3*

2. How many triangles in a hexagon? _____

3. What is the minimum number of conditions needed to construct unique hexagons? _____

4. Why?

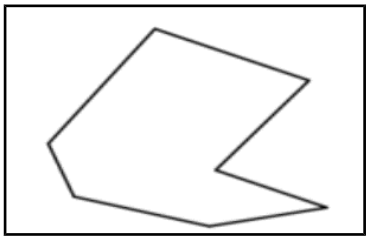

*Figure 4*

5. How many triangles in a heptagon? _____

6. What is the minimum number of conditions needed to construct unique heptagon? _____

7. Why?

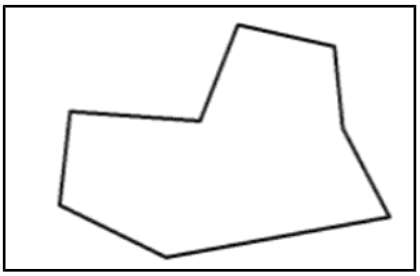

*Figure 5*

8. How many triangles in an octagon? _____

9. What is the minimum number of conditions needed to construct congruent octagon? _____

10. Why?

## References


- http://www.amesa.org.za/amesal_n21_a12.pdf
- https://www.mathopenref.com/congruentpolygons.html
- NCERT Mathematics textbook Class 7, Chapter 7; Class 8, Chapter 4