Mid-Point Quadrilaterals

Overview:
In this unit students will use GeoGebra, an open source dynamic geometry software to explore and make conjectures regarding the midpoint quadrilateral theorem. They will also look at some special cases of the mid-point quadrilateral theorem and prove the special cases.

Minimum time required:
Minimum 2 sessions of 40 minutes (After the students have practiced GeoGebra)

Type of LU:
Computer Laboratory

Link to the curriculum
Chapter 3 - Class 8
Chapter 8 – Class 9

Introduction:

In this learning unit, you will explore quadrilaterals formed by joining the midpoints of any given quadrilateral using the software GeoGebra. And you will also use Geogebra to make conjectures about special cases of quadrilaterals. And also prove them.

Learning Objectives:
Exploring the properties of quadrilaterals using GeoGebra
Conjecturing, Verifying and Proving the mid-point quadrilateral theorem
Conjecturing the special cases of the mid-point quadrilateral theorem and proving them

Prerequisites:
Familiarity with the basic construction tools in GeoGebra
Properties of special quadrilaterals

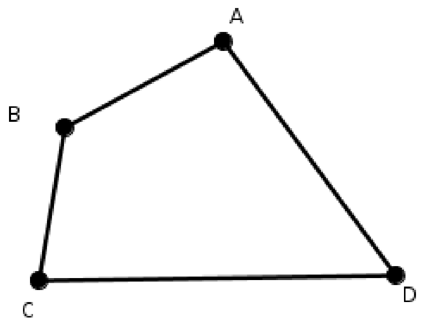
Materials Required:
Computers (students may work in pairs) with GeoGebra installed, Papers, Pencils/Pens

Suggested Readings
https://archive.geogebra.org/GeoGebra-in10Lessons.pdf
https://www.youtube.com/watch?v=1cBXWi66-tY
(*A Geogebra Manual*) (*A GeoGebra Tutorial*)
https://demonstrations.wolfram.com/TheMidpointQuadrilateralTheorem/
https://mathforlove.com/2012/02/midpoints-of-a-quadrilateral-form-a-parallelogram/
https://www.techhouse.org/~mdp/midpoint/nonquad.php

Sketch and Investigate

Open a new page in GeoGebra. Click on View and then on Axes to hide the axes. Only the algebra and graphics views should be visible.

Task 1:

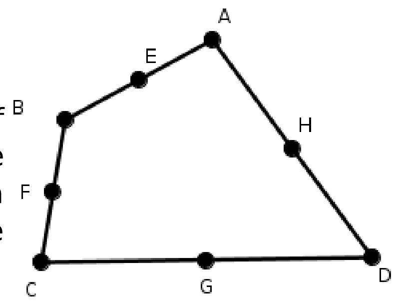


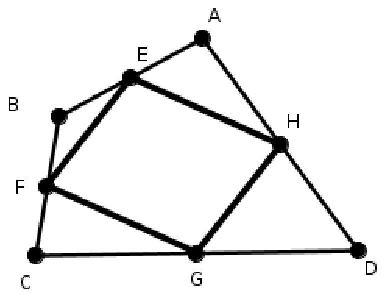
Construct a quadrilateral ABCD using the Polygon Tool. To do this click on the **Polygon tool** and click the points in the following order: point *A*, point *B*, point *C, point D* and then point *A* to close the polygon. Note that GeoGebra labels the vertices with uppercase letters and the line segments with small case letters. Right click on each of the sides of the quadrilateral ABCD and select Show Label from the menu to hide the labels a,b,c and d of the sides.

To avoid cluttering the figure with too many labels, go to Options in the main menu, select Labeling followed by New Points Only. This will ensure that new line segments are not labeled.

Task 2:

Find the midpoints of the sides, AB, BC, CD and AD. Some of them are marked here. This can be done by selecting the Midpoint or Center icon from the Point Tool menu and then clicking on the four sides of the quadrilateral ABCD. The midpoints will be labeled as E,F G and H respectively.





Task 3:

Make the new quadrilateral by selecting the Polygon tool and then clicking on the points E,F G and H in that order . This will be called the midpoint quadrilateral.
By a mid-point quadrilateral we mean a quadrilateral formed by joining all the midpoints of the sides of a given quadrilateral.

Task 4:

Drag the vertices of the original quadrilateral ABCD and observe what happens to the midpoint quadrilateral EFGH. Record your observations below.
Is there any similarity between all the mid-point quadrilaterals you got while dragging the vertices?

> If the students are unable to notice that e = g and f = h bring it to their attention but before doing so give them enough time.

Note the lengths of the four sides of quadrilateral EFGH (which are marked as e, f, g and h) in the Algebra view. What do you observe? Now drag one of the vertices of the original quadrilateral ABCD. What are your observations regarding e, f, g and h? Based on the observations, what can you say about them?

Task 5:

What kind of quadrilateral do you think EFGH is? Use your observations to support you conjecture?

> In the above task, the students are asked to look at lengths of opposite sides of the mid-point quadrilateral, another way to check that the quadrilateral EFGH is a parallelogram is checking the angles of the quadrilateral using the angle tool.
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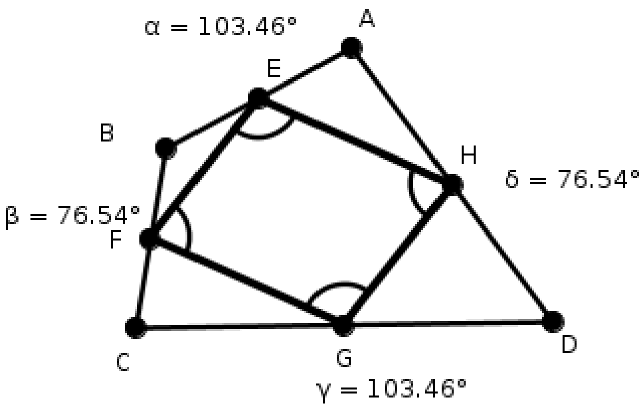
> You can ask the students to drag the vertices to check if both the pairs of opposite angles are equal or not.

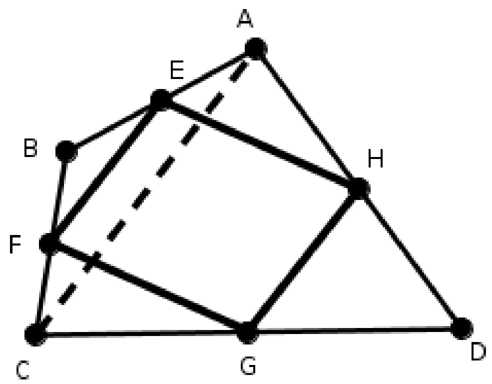


> You can also ask the students in which all quadrilaterals that happens.
> The students can observe different aspects of the mid-point quadrilateral which points out that it is a parallelogram. Some of them are
>
> 1) Both pairs of opposite sides are equal
> 2) Both pairs of opposite angles are equal
> 3) The diagonals bisect each other
> All of these characteristics can be verified using Geogebra.

Task 6:

The diagonal AC divides the quadrilateral ABCD into two triangles. In each of these triangles one of the sides of the midpoint quadrilateral is a mid segment (segment joining the midpoints of the other two sides).

Use this information to validate the conjecture made by you.



> At this stage some students might say that EFGH is a trapezium. Use this opportunity to discuss the definition of trapezium and ask them what will happen if they look at diagonal BD.

Task 7:

Would you be able to make similar conjectures by considering the diagonal BD of quadrilateral ABCD instead of diagonal AC?

Task 8:

Observe the numbers associated to Poly1 (ABCD) and Poly2 (EFGH) in the Algebra view. Can you see a relationship between them?

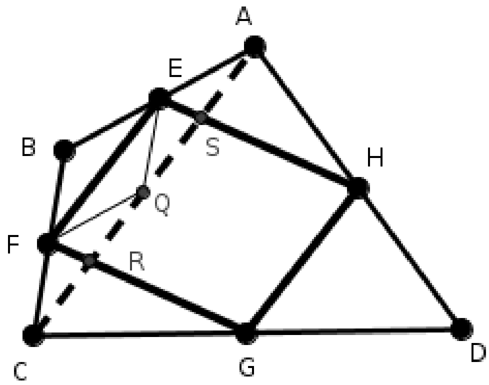
From this, what can you say about the area of the midpoint quadrilateral EFGH in relation to the area of ABCD?

Task 9:

Prove your conjecture.

For Tasks 8 and 9

Here EFGH is the midpoint quadrilateral of the quadrilateral ABCD.



Q is the mid-point of diagonal AC. And points R and S are intersecting points of AC with FG and EH respectively.

Now let us look at triangle ABC,

Area of FCQ = Area EQA = Area of BEF = Area of EFQ
(Base and height of the four triangles are equal.)

So,
Area of EFQ = 1/4 (Area of triangle ABC)
Similarly, Area of DGH = 1/4(Area of triangle ACD)
Also, if you draw the diagonal BD, then

Area of AEH = 1/4 (Area of triangle ABD)

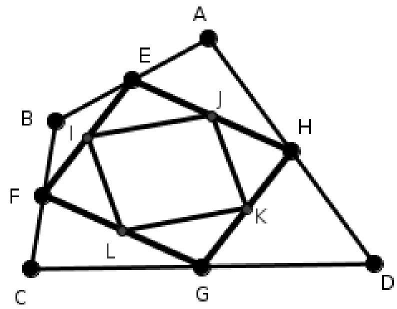
Similarly, Area of CFG = 1/4(Area of triangle BCD)So,
Area of EFQ + Area of DGH + Area of AEH + Area of CFG
= 1/4 (2 x Area of ABCD)
= ½ (Area of ABCD)
Area of EFGH
= Area of ABCD - (Area of EFQ + Area of DGH + Area of AEH + Area of CFG)
= ½ (Area of ABCD)

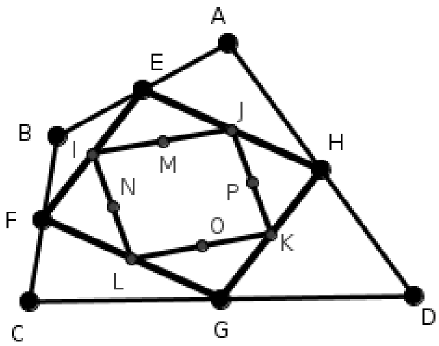
So Area of Parallelogram EFGH = 1/2 (Area of Quadrilateral ABCD).

You can ask the children to observe the poly 1(ABCD) and poly 2 (EFGH) in the Algebra view and ask them whether they see a relationship between these two.

Task 10:

Draw the midpoint quadrilateral of EFGH? GeoGebra will label it as IJKL. What kind of a quadrilateral is IJKL? How is its area related to that of EFGH and ABCD?





Task 11:

Continue drawing midpoint quadrilaterals as you did in 8. Can you see how these midpoint quadrilaterals are related to each other?

> For the tasks given below ask the students to discuss the ways they would construct a square, a rectangle or a rhombus using GeoGebra. You can help them by asking them to recall various properties they know of these quadrilaterals. The parallelograms that the students will get in the constructions below will be all of the special kind. Inside the square, they will get a square. Inside a rectangle, they will get a rhombus and inside a rhombus they will get a rectangle. Encourage them to make these conjectures and prove them. Also highlight the fact that using GeoGebra they are only verifying their conjectures and not proving them.

Task 12:

Using GeoGebra, draw a rectangle. Move the vertices of the rectangle you have drawn and check if the rectangle remains a rectangle. If not, draw again.

Write down the steps you took to ensure that you have drawn a rectangle.

Task 13:

Draw a mid-point quadrilateral of this rectangle. What can you say about this mid-point quadrilateral? Can you prove your conjecture?

> Proof of the statement: “Midpoint Quadrilateral of a rectangle is a rhombus”
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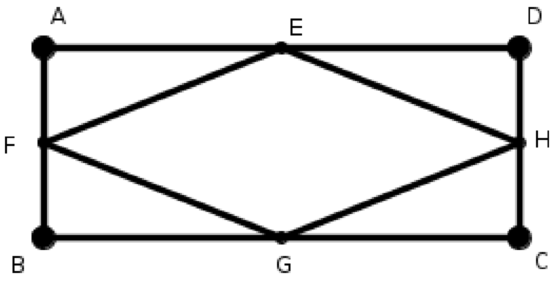
> 
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>
> Given that ABCD is a rectangle
> And E, F, G and H are midpoints of the four sides AD, AB, BC and CD respectively
> So, AE = ED = BG = CG
> And, AF = FB = DH = CH
> Also, the four triangles (AEF, DEH, CGH and BFG) are right angle triangles,
> Hence,
> EF = EH = HG = FG (Pythagoras Theorem)
> Hence EFGH is a rhombus

Task 14:

Using GeoGebra, draw a rhombus. Move the vertices of the rhombus you have drawn and check if the rhombus remains a rhombus. If not draw again.

Write down the steps you took to ensure that you have drawn a rhombus.

Task 15:

Draw a mid-point quadrilateral of this rhombus. What can you say about this mid-point quadrilateral? Can you prove your conjecture?

Task 16:

A square is both a rectangle and a rhombus. Then what can you say about the midpoint quadrilateral of a square.

> **Additional Tasks:**
> What can you say about mid-point quadrilaterals for trapeziums? What about isoscles trapeziums?

References:

Lingefjärd, T., Ghosh, J, Kanhere, A. (2015). Students Solving Investigatory Problems with GeoGebra - A Study of Students’ Work in India and Sweden. In S.J. Cho (Ed.),
The Proceedings of the 12th International Congress on Mathematical Education