An Experiment on Measuring Volumes

Overview

The childhood story of the crow and the urn is very popular among kids. Using this simple story we can actually explain the concept of volume to the students. In this learning unit, students will learn how to estimate volumes of different bodies by submerging them in water. We also introduce the idea of packing fraction of solids, i.e. when you try to pack together many irregular solid objects, there are always some gaps in between and 'the crow cannot raise level of the water' beyond a certain limit.

Minimum Time Required: 3 sessions of 40 mins.

Type of Learning Unit: Classroom Activity

Learning Objectives

(i.) This activity teaches the student how to make a marked glass cylinder and use it to measure the volume of liquids. The student learns how to use the idea of fluid displaced by submerged bodies to measure volume of solids that do not dissolve in water. The activity also teaches about the importance of least count of a measuring instrument and it gives an idea about how accurate her volume measurement is.

(ii.) At the end, the student is posed a challenge question. If the student understands the answer, it will give an intuitive understanding of the concept of packing fraction.

Learning Physics From the Crow and the Pitcher Story

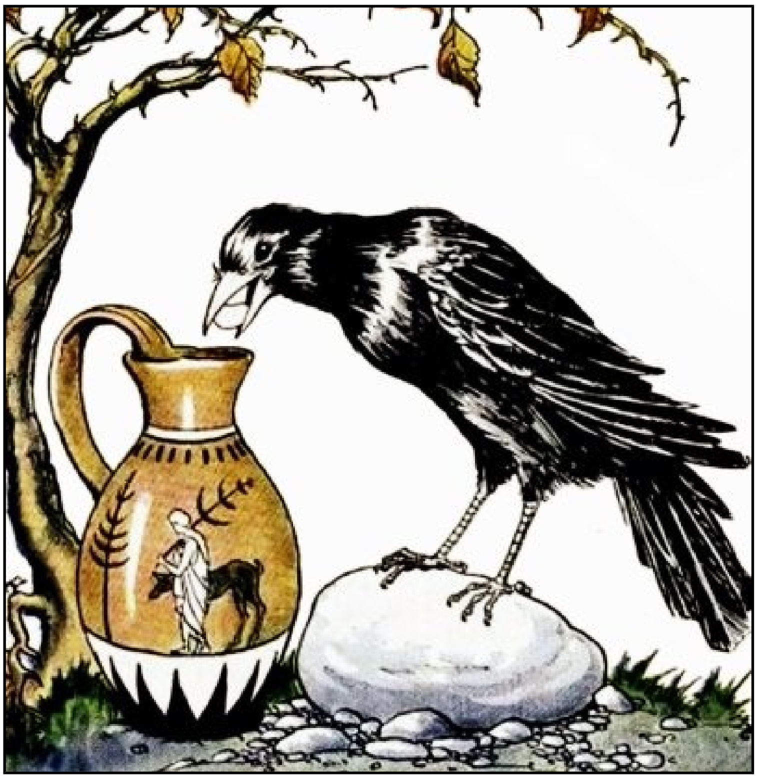

Figure 1: Borrowed from The Aesop for Children, by Aesop, illustrated by Milo Winter, Project Gutenberg etext 1994

Do you remember the childhood fable of the crow and the pitcher? If you have forgotten, this may jog your memory.

In this unit, we will imitate the crow in the story and use the scientific principle that a body submerged in water displaces an amount of water equivalent to its volume to carry out some measurements. The last task is actually very closely related to the tale– and you may reach a surprising conclusion at the end of it!

Materials Required

- A narrow transparent cylinder (or a transparent 500 mL water bottle with the top cut off).
- Similar sized glass marbles (~40)
- Small irregular stone which can fit into cylinder comfortably (see note in task 4)
- Ruler
- Marker pen that draws thin mark
- Straight edge (like another ruler or edge of a notebook)

> **Are you familiar with these ideas?**
> - Volume:
>   Students should be familiar with the concept of volume in general and also the formula for volume of a sphere.
> - Displacement of fluids by solid objects:
>   Students should be aware that when a body is submerged in a fluid, it displaces its own volume of the fluid. Familiarity with the Archimedes principle of buoyancy is not necessary for this task.
> - Average/ Mean:
>   Students should be familiar with the concept of average of several quantities.

Task 1: Creating Your Own Volume Measuring Instrument (a Graduated Cylinder)

1) Use the beaker to carefully measure 50 ml water and transfer it to the transparent cylinder. Mark the height of the water on the cylinder using a marker pen.
2) Repeat this till the cylinder is almost full, marking successive heights at steps of 50 ml.
3) Label the markings with appropriate multiples of 50 ml. (50,100, 150,...)

Now, you have a graduated cylinder which can measure volume. You will notice that we can use this cylinder to measure volume only in multiples of 50 ml. Hence 50 ml is the least count of this graduated cylinder.

Maximum volume your graduated cylinder can measure ________________________
(Highest marking on the cylinder)

> - The teacher should explain the concept of least count to the students. In this context, the least count would be 50 ml. Note that here we are assuming that whenever the water level is in between two marks, we will take the smaller of the two values as the reading.
> - We will see later in task 4 how to increase the accuracy of this graduated cylinder beyond the current least count.
> - The graduated cylinders prepared by the students in this task can also be reused for other experimental purposes.

Task 2: Measuring the average volume of marbles

1) Take the empty cylinder and fill it up to 200 ml mark.
2) Drop marbles in the transparent cylinder one by one while counting them, until the water level rises up to the next mark. The water level rises because each marble displaces an amount of water equal to its own volume.

   Volume of water before adding marbles ________________________________

   Volume of water after adding marbles ________________________________

Number of marbles required to raise the water level to the next mark _______________
Thus, _______ marbles displace __________ volume of water.

3) Use this result to estimate the average volume of one marble. (V_{exp})

Average volume of one marble ________________________

- It is possible that the students may not be able to get the water level exactly up to a mark on the transparent cylinder. The teacher should explain to the students that if the water level falls below the nearest mark for say 20 marbles and goes above the mark for 21 marbles, then in such case, number of marbles should be considered as 20.
- The teachers may explain that the students obtain the volume for “n” marbles in this task and then find the average volume of one marble. Note that all the marbles are nearly equal in volume, but not exactly equal. This is why it makes sense to take an average to find volume of one marble.

Figure 2: A measuring cylinder with water and marbles

- When we performed this task, the result we obtained for the set of marbles that we chose was 25 marbles ~ 50 ml i.e. average volume of one marble ~ 2 ml.

Task 3: Comparing the measured volume of marble with that obtained by using formula

1) Keep 10 marbles in a straight line touching each other (You can create a long narrow channel by a straight edge on one side and ruler on other side).
2) Measure the end to end length of the line of marbles.

   End to end length of marbles ____________________________
3) Use this measurement to estimate the average radius of the marbles.

   Average radius of one marble ____________________________
4) Calculate the volume of a sphere $\left(V_{math}=\frac{4}{3}\pi r^3\right)$ using the radius you have obtained.

   Volume of one marble ____________________________
5) Estimate the percentage error using the formula: $\frac{|V_{math}-V_{exp}|}{V_{math}}\times 100$ %

   Percentage error = ______________________________

- Dividing the end to end length by the number of marbles will give the mean diameter of the marbles. The teacher may have to explain this point, in case the students fail to appreciate it.

- For our set of marbles, the end to end length of line of marbles was 10 marbles ~ 15.1 cm. i.e. 1 marble ~ 1.51 cm. This gives the volume of one marble as $\frac{4}{3}\pi r^3 = (\frac{4}{3}) \times (\frac{22}{7}) \times (\frac{1.5}{2})^3$ = 1.77 ml.
- The teacher may have to explain the concept of error qualitatively as follows: There is a clear discrepancy in the volume of one marble obtained in task 2 and that obtained in task 3. This is because in task 2, we are calculating the average volume of one marble directly without finding the radius of marbles. On the other hand, in task 3, we find the average radius of marbles and then calculate the volume. In this process, any error in the measurement of the radius gets cubed (due to $\frac{4}{3}\pi r^3$) and hence gives a larger error in the volume. This error can be minimised by increasing the number of marbles taken to measure the end to end length and then calculating the corresponding average radius of the marbles.
- Possible extension: If the students want, they can repeat this with more number of marbles and/or with more random sets of 10 marbles. Will the answer move closer to task 2 in this case?
- Possible extension: Students may also try repeating the tasks 2 and 3 using machined steel balls (ball bearings) of size similar to that of marbles. Since these are usually manufactured to higher degree of precision than the marbles, the discrepancy in thevolume obtained in task 2 and that obtained in task 3 will be smaller.

Task 4: Measuring the Volume of an Irregular Stone

1) Fill the cylinder to 200 ml mark with water.
2) Put an irregular stone in the water (The stone should be completely immersed inside the water with the water level at least 2-3 cm above the upper surface of the stone).
3) Estimate the volume of the stone by observing the amount of water displaced. Unlesso the water level matches with one of the markings, this will only be approximate measurement.
4) Now, immerse enough marbles to bring the water level up to the next marking.
   Volume of water before adding the stone ______________________________
   Number of marbles required to raise the water level to the next mark
   ____
   The irregular stone + ______ marbles displaced _______ volume of water.
   Final volume f stone + marbles + water ______________________________
5) Use the mean volume of marbles, V_{exp} obtained in task 2 to determine the volume of the stone more precisely.
   Volume of the irregular stone ______________________________

- Please note that once the stone is submerged, there is sufficient depth of water on top of the stone to immerse a few marbles completely. If this is not the case the teacher may consider restarting with a larger amount of water.
- The teachers should emphasise that the mean volume of marbles is obtained here to a greater accuracy than the accuracy imposed by the least count of our "manufactured" cylinder.

Task 5: A challenge

1) Fill up the cylinder to 50 ml mark.
2) By adding enough marbles, try to raise the **water level** to the top of the cylinder.
3) If you do not succeed in raising the water level to the top, can you estimate the

maximum marking to which the water level rises?

Maximum marking to which the water level rises ____________________________

Number of marbles required to raise the volume by that much amount __________________

4) Can you think of an explanation for this?

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5) Do you think the thirsty crow would have succeeded in quenching its thirst?

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- The student is expected to fail to bring the water level up to the top. Once this happens, the teacher may explain to them that when objects like marbles are packed as closely as possible, they still have some gaps in between and the water would accumulate there and hence will never rise above a certain multiple of the original level. Assuming closest possible packing of the marbles, this multiple is ~4. Thus, in this task the water level is not expected to rise beyond 200 ml. The teacher may also point out to the more interested students that this concept is called "packing fraction" and is important in the theory which explains atoms are packed together in solids.
- Thus, the upshot of the story is the thirsty crow would not have succeeded in quenching its thirst if the water level in the pitcher had been too low to begin with.
- Only for the teachers: In case of marbles, the closest possible packing is called "hexagonal close packed". The packing fraction for this is $\frac{\pi}{3\sqrt{2}}$ ~ 0.74. This means that 74% of the volume will be occupied by marbles and at most 26% by water. Further, on the edges of the cylinder, this fraction will be poorer. Hence, we expect water to be closer to 150 mL.
- It might be worth expanding the analogy to explain that groundwater is also the water that is filled in the pore spaces of soil particles and stones beneath ground. Thus even though pore spaces in soil seems tiny, they hold the huge amounts of groundwater that we extract via handpumps and borewells.
- Can you calculate how much groundwater can be filled in a 10 m x 10 m area to 1 m depth if it is made of spherical soil particles of 1 mm diameter? (Hint: calculate the volume of the total region and subtract the volume of all the particles in that region.)

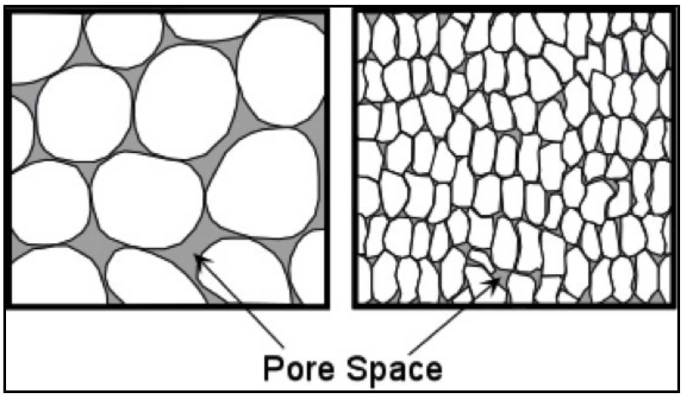

Figure 3: Pore space between soil particles

- Hexagonal close packing you may have observed in stacking of balls or of fruits in the market.

Figure 4: Oranges stacked in a fruit stall

Idea for Possible Extension

If the students have either different diameters for bottles or different sizes of marbles or both, what is the maximum marking to which the water level rises?

Suggested Readings

1. https://wwwrcamnl.wr.usgs.gov/uzf/abs_pubs/papers/nimmo.04.encyc.por.ese.pdf (soil and pore space)
2. http://www.nzdl.org/gsdlmod?e=d-00000-00---off-0cdl--00-0----0-10-0---0---0direct-10---4-------0-1l--11-en-50---20-about---00-0-1-00-0--4----0-0-11-10-0utfZz-8-00&cl=CL1.136&d=HASH3105beaa24976abbef784e.5.2&gt=1(soil and pore space)
3. https://www.thoughtco.com/archimedes-volume-and-density-3976031 (Archimedes story)