Maps, Time-Tables and Radio Stations: What is common in all?
-Mathematical Modeling

Materials required:

Worksheets, Pens, Pencils, Erasers, Color Boxes, Board, Projector

Recall:

- Have you already completed the Koenigsberg Bridge Learning?
- Are you comfortable with basic definitions of vertex and edges?

Task 1 : Realistic problem

The king and his kingdom

Once upon a time, there was a kingdom ruled by a king who had three daughters and two sons. It was his wish that upon his death, this kingdom should be divided into five regions, one region for each child, such that everyone is everyone's neighbour, that is each region would have a common boundary with every other regions.

What do you think, can this be done? Think about and discuss the solution with your partner. _____
_____

Think about how you can simplify your figure. Remove the details which not required to solve the problem? Draw a simplified version of your figure and discuss with your partner how your picture/diagram still represents the problem given.

Task 2 : Colouring a Map

Have you ever saw a coloured map of India? What have you noticed in it? Students might response states, districts, sea and so on.
If you see any map in the atlas or on the Internet (e.g.https://www.mapsofindia.com/), you will notice that in every map, regions which are next to each other; they share a boundary are always colored in different colours so that the regions next to each other can be distinguished. Let us try to explore this idea.

Q. What is the minimum number of colours you have observed in any map?_____

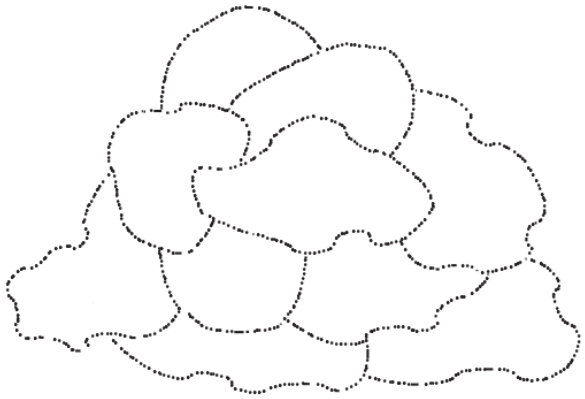
Colouring 1:
We are giving you a map, can you colour it in such a way that every marked region can be clearly distinguished from its neighbours. Try to use minimum number of colours as possible.

Try to estimate the number of colours needed before actually starting to colour and then do actual colouring.

Q. What is the minimum number of colours you needed? _____

Colouring 2

Try to estimate the number of colours needed before actually starting to colour and then do actual colouring.



Q. How many minimum colours you actually need to colour the given map? _____

Can you convert this map into graph?

Q. What do you think how many colours you will need to colour the vertices? _____

Task: 3 Time-table problems

A school is planing to start Vigyan Pratibha sessions for their Class 8 students. There are four different themes of subjects.

This is not easy! Because some students are taking interest in several subjects, and a student can attend only one session during a particular time slot. The school wants to avoid all conflicts. Of course, you can make such a schedule by having every sessions in a different time slot, but then you would need four periods in current timetable for the four Vigyan Pratibha themes, and the session would run all the day!

The teachers decided that sessions can be categorized according to 4 themes which is, Mathematics, Physics, Chemistry and Biology. The school asks the participating students to pick maximum two of the themes. After collecting choices of all students, teachers saw following different combinations:

Mathematics – Physics, Mathematics – Biology, Physics – Chemistry and Biology – Chemistry.

What is the minimum number of periods that would be allotted for these sessions so that every student can attend the sessions of their choice. Time-table came and on the very first day there were two parallel class of Physics and chemistry. Shama and two of her friends Rose and Amar chose Physics and Chemistry combination, but they couldn't attend the Physics class and Shama pointed out this to teacher immediately. Hence teacher has to make timetable again.

Can you help the teacher to arrange the timetable?

What if...
(I) There are a lot of children who want Chemistry and Mathematics both. The school accepted it and now the new combinations were:

Mathematics – Physics, **Mathematics – Biology**, **Physics – Chemistry**, **Biology – Chemistry** and **Mathematics – Chemistry**

Then what will be the minimum number of periods that would be allotted for these sessions so that every student can attend the sessions both the subjects of their choice?

Now it's your turn,

What if… In the new term, the school decided to add another subject, Local Context to the list. So the new combinations were:

Mathematics – Physics, **Mathematics – Biology**, **Physics – Chemistry**, **Biology – Chemistry** , **Mathematics – Chemistry and Biology – Local Context.**

Now, what is the minimum number of periods that would be allotted for these sessions so that every student can attend the sessions for subjects of their choice?

Task 4: Real life situation

Have you heard any FM channels on the radio? Can you name some of these channels? In Mumbai, All India Radio (AIR) has several FM channels. Usually, some number is used to on dial of any FM radio to tune to any FM channel. e.g AIR's FM Gold 100.1, FM Rainbow 107.1 and FM Vividha Bharati 102.8. Do you know what these numbers mean? If you don't know the answer, discuss with your friends and teachers.

Have a discussion in the class about radio frequencies and how different stations run at different frequencies. Then ask them, "What will happen if two radio stations in neighbouring cities run (say Mumbai and Pune) at the same frequency?" If students struggle to come up with an answer, prompt them to think what will happen at midpoint of the two cities (say Lonavala). If some students answer saying that there will disturbance / the radio handset will get garbled mix of both stations, then ask them to tell the class why. When radio handset receives two signals at the same frequency, the listener would not be able to hear anything clearly.

Now let us attempt the problem of assigning broadcast frequencies to different radio stations. If we have too many different frequencies, then our handsets should be equipped to accepts frequencies from a wider range (bandwidth), which makes them more expensive. Thus, it is in our interest to allot as fewer frequencies as possible to the FM channels. At the same time, we must ensure that neighbouring radio stations should not get same frequency. What do we do? Again, let us start with a simple example:

Q. Let us start with say 5 radio stations R1, R2, R3, R4 and R5. Among these, R1 and R2 are neighbours. R2 is also a neighbour of R3 as well as R4. Further, R3 is a neighbour of R5. Given these conditions, how many minimum frequencies would be needed to ensure that there is no interference?

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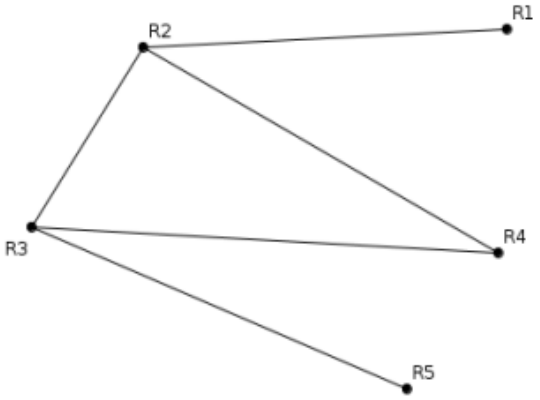
Radio Station	Neighbouring Radio Stations
R1	R2
R2	R1, R3, R4
R3	R2, R4, R5
R4	R2,R3
R5	R3



If we put the radio station as vertices and join the neighbouring stations with edges, the picture would look something like this.

Can you colour these vertices like previous task? In this case, number of colours would mean number of frequencies needed.

Q Can you construct a new situations and give your partner/friend to determine the minimum number of bandwidths required