Finding the Right Path

Task 1: Seven Bridges of Konigsberg!

Today we are going to begin with the story of Konigsberg in the 18th century, its geography, bridges, and the question asked by its citizens.

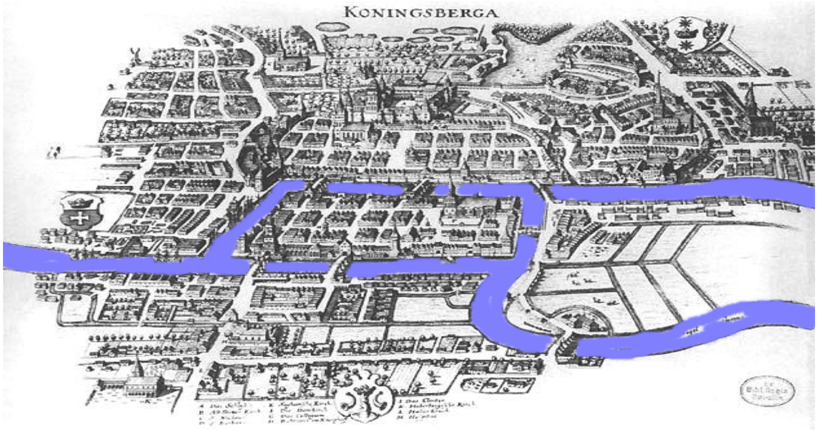

Figure 1

You may remember the story of Alexander Fleming's discovery of penicillin in 1928. He noticed that a bacteria culture left open by mistake had been infected by a mold that appeared to inhibit the growth of the bacteria. Thus began our age of antibiotics. This episode demonstrates how a small event, when viewed in a new way by an alert mind, can lead to dramatic advances in our knowledge.

The history of Mathematics too has a famous story of this type, connected to the city of Kaliningrad in Russia. Kaliningrad lies between Lithuania and Poland and is at some distance from the rest of Russia. In fact, it was originally a German town and was called Konigsberg. The river that runs through this town was then called the River Pregel. The Pregel branched and looped through Konigsberg, as shown in the picture below, and in the eighteenth century there were seven bridges across it.

A challenge took shape around the river and the bridges – it was to find a path that would let one walk across all seven bridges exactly once. No bridge could be missed or crossed twice and, of course, there was to be no swimming across the river!

Can you state the problem of walking over the 7-bridges in your own words? Is this a mathematics problem?

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Look at the following picture.

Figure 2

Is this picture same as the one you saw of Konigsberg Bridges? Why do you think so?

Think about further simplifying this picture. Remove the details not required to solve the problem? Draw your simplified pictures here, and discuss with your partner how your picture/diagram still represents the problem of 7-bridges of Konigsberg.

Like all of you the citizens of Konigsberg were also not able to solve the problem. Their Mayor wrote to the famous mathematician Leonhard Euler for help. And the first thing Euler did was to create a simplified and labeled drawing, just as done by the students in class. Here is his drawing:

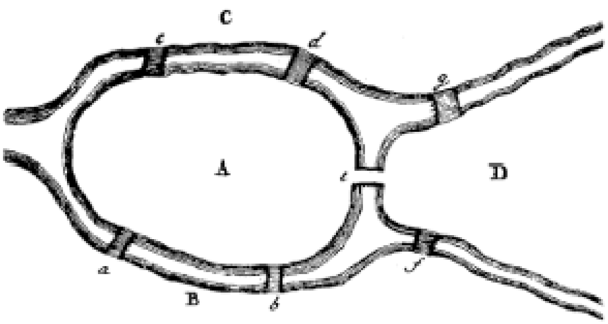


Figure 3

Look at this map that Euler made. How do you make sense of it? What are those letters?

Now you can label the path. One example is “*cabdgebf*”. You can see that this path used the bridge *b* twice. So this is not a required path. Can you find the required path? Share your paths with your friends.

Do you remember the popular childhood puzzle about drawing a square and its diagonals without lifting the pencil or retracing any part?

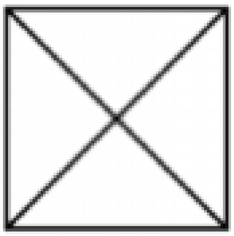


Figure 4

Were you successful? How did you do it?

Task 2: Graphing the Reality!

See the following graph. This graph represents the same 7-bridges problem that we were working on till now. Explain how it is the same problem. Where are the rivers and lands?

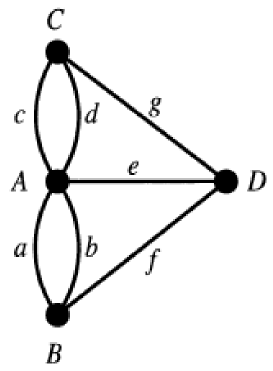
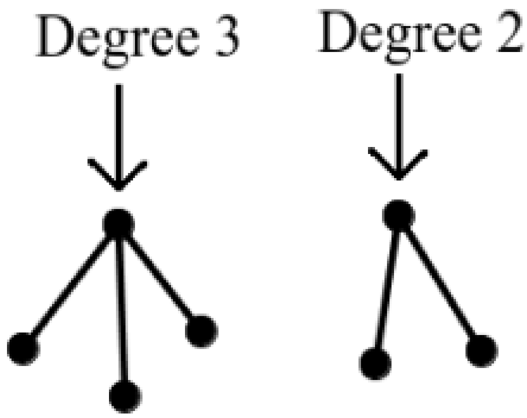


Figure 5

Can you trace the entire graph above without lifting your hand? Now this problem is same as the problem citizens of Konigsberg came across – waking over all the bridges once. Try here, and try with different starting points.

A graph in Graph Theory consists of edges and vertices. The graphs are diagrams where there are vertices and lines joining any two vertices are called edges.

The number of edges, that join at a vertex are called as degree of the vertex.



The number of edges that lead to the vertex is called the degree of that vertex

Figure 6

For example the following diagram has 6 vertices and 7 edges, vertices A, C and E have degree 3; vertices D and F have degree 2 and vertex B has degree 1.

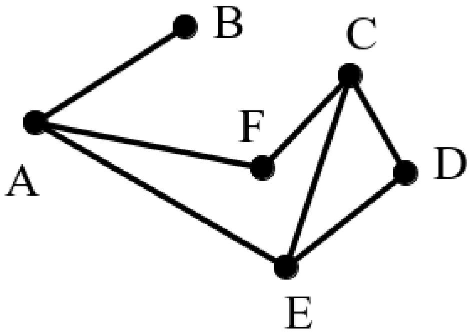


Figure 7

Draw a graph of your own; describe degree of its vertices.

Study the following graphs. In each of them, see whether it is possible to find a path that passes through every edge without repetition. Try different vertex as starting points.
Label the graphs, so that you can describe the path as sequence of letters.

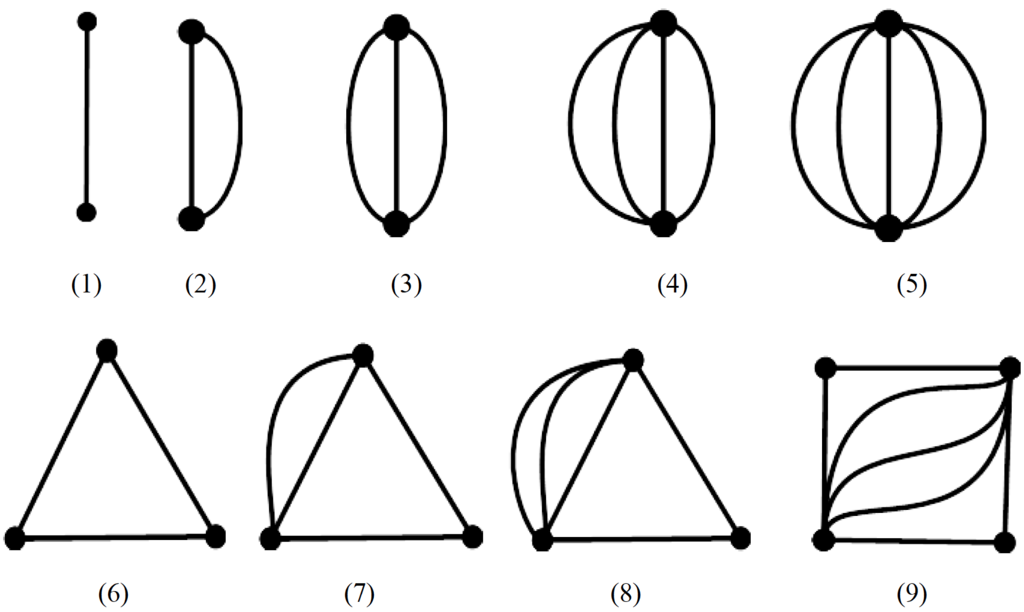


Figure 8

Here is one example labeled and explained, the Figure 9.

	Path	First vertex = Last vertex (Yes/No)	Degree of first vertex	Degree of last vertex	Degree of other vertices
7	ABCAB	No	3	3	C --- 2

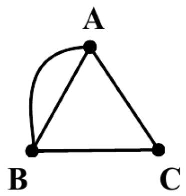


Figure 9

Record your findings for each graph in the following table:

Graph No.	Path	First vertex = Last vertex (yes/no)	Degree of first vertex	Degree of last vertex	Degrees of other vertices
1					
2					
3					
4					
5					
6					
7	ABCAB	No	3	3	C - - - 2
8					
9					

Study the pattern carefully in the table and write your guesses about what features of the graph makes the graph traceable (crossing the each edge only once) without lifting your hand.

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What pattern do you see for the graphs where the starting and ending point of the path is the same vertex? Write statements of your conjectures.

How do you know the statements you made are true?

References:

[BTM] Shobha Bagai, Amber Habib, Geetha Venkataraman, A Bridge to Mathematics, SAGE India, 2017.

[Edkins] http://gwydir.demon.co.uk/jo/games/puzzles/bridge.htm An online game where a figure actually walks across the bridges.

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