Exploring Patterns in Square Numbers

Introduction

Task 1: The following are the squares of numbers from 1 to 20. What patterns do you see?

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400

Task 2:

Given below is a number chart. Shade the squares that have square numbers. The first few are done for you. What patterns do you notice?

I	II	III	IV	V	VI	VII	VIII		I	II	III	IV	V	VI	VII	VIII
1	2	3	4	5	6	7	8		209	210	211	212	213	214	215	216
9	10	11	12	13	14	15	16		217	218	219	220	221	222	223	224
17	18	19	20	21	22	23	24		225	226	227	228	229	230	231	232
25	26	27	28	29	30	31	32		233	234	235	236	237	238	239	240
33	34	35	36	37	38	39	40		241	242	243	244	245	246	247	248
41	42	43	44	45	46	47	48		249	250	251	252	253	254	255	256
49	50	51	52	53	54	55	56		257	258	259	260	261	262	263	264
57	58	59	60	61	62	63	64		265	266	267	268	269	270	271	272
65	66	67	68	69	70	71	72		273	274	275	276	277	278	279	280
73	74	75	76	77	78	79	80		281	282	283	284	285	286	287	288
81	82	83	84	85	86	87	88		289	290	291	292	293	294	295	296
89	90	91	92	93	94	95	96		297	298	299	300	301	302	303	304
97	98	99	100	101	102	103	104		305	306	307	308	309	310	311	312
105	106	107	108	109	110	111	112		313	314	315	316	317	318	319	320
113	114	115	116	117	118	119	120		321	322	323	324	325	326	327	328
121	122	123	124	125	126	127	128		329	330	331	332	333	334	335	336
129	130	131	132	133	134	135	136		337	338	339	340	341	342	343	344
137	138	139	140	141	142	143	144		345	346	347	348	349	350	351	352
145	146	147	148	149	150	151	152		353	354	355	356	357	358	359	360
153	154	155	156	157	158	159	160		361	362	363	364	365	366	367	368
161	162	163	164	165	166	167	168		369	370	371	372	373	374	375	376
169	170	171	172	173	174	175	176		377	378	379	380	381	382	383	384
177	178	179	180	181	182	183	184		385	386	387	388	389	390	391	392
185	186	187	188	189	190	191	192		393	394	395	396	397	398	399	400
193	194	195	196	197	198	199	200		401	402	403	404	405	406	407	408
201	202	203	204	205	206	207	208		409	410	411	412	413	414	415	416

Task 3A1:

Predict in which column the following will appear without evaluating? Justify your answer. How do you know for sure?

a) 75^2 b) 96^2, c) 122^2, d) 100000^2 e) 12346^2

Task 3A2:

Is it possible that 12345672 is in column 5? Why do you think so?

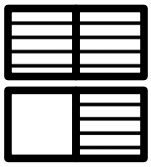
Task 3A3:

Prove at least 4 of the patterns that you came up with in Tasks 1 and 2.

Task 3B:

Here is a square.

We can add three more squares to make it into a 2 x 2 square.



2

Build on this to make a 3 x 3 square. How many squares did you have to add?
Now use a 3 x 3 square to make a 4 x 4 square. How many squares did you have to add?

Continue the process for a few more steps. What patterns do you observe? Is it related to any of the patterns that you noticed earlier? Can you prove the pattern?

Task 3C

Task 3C1 :

Plot the last digit of the square of a number against the number. What patterns do you see?

Task 3C2

Notice that the pattern of last digits 1, 4, 9, 6, 5, 6, 9, 4, 1 0 repeats.
Can you prove that the pattern of last digits repeats?

Task 3C3

Instead of the grid in Task 1 if we were to have a 10 column grid like the following, and we shade the squares with perfect squares, can you guess in which columns these numbers would be? Are there columns where there would be no shaded numbers?
If you were to divide a perfect square by 10, what possible remainders would do you get?

	II	III	IV	V	VI	VII	VIII	IX	X
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180

Task 3C4:

Guess what patterns would you see if the numbers were written in a 3 column grid. Where would the square numbers be?

I	II	III		I	II	III		I	II	III		I	II	III
1	2	3		61	62	63		121	122	123		181	182	183
4	5	6		64	65	66		124	125	126		184	185	186
7	8	9		67	68	69		127	128	129		187	188	189
10	11	12		70	71	72		130	131	132		190	191	192
13	14	15		73	74	75		133	134	135		193	194	195
16	17	18		76	77	78		136	137	138		196	197	198
19	20	21		79	80	81		139	140	141		199	200	201
22	23	24		82	83	84		142	143	144		202	203	204
25	26	27		85	86	87		145	146	147		205	206	207
28	29	30		88	89	90		148	149	150		208	209	210
31	32	33		91	92	93		151	152	153		211	212	213
34	35	36		94	95	96		154	155	156		214	215	216
37	38	39		97	98	99		157	158	159		217	218	219
40	41	42		100	101	102		160	161	162		220	221	222
43	44	45		103	104	105		163	164	165		223	224	225
46	47	48		106	107	108		166	167	168		226	227	228
49	50	51		109	110	111		169	170	171		229	230	231
52	53	54		112	113	114		172	173	174		232	233	234
55	56	57		115	116	117		175	176	177		235	236	237
58	59	60		118	119	120		178	179	180		238	239	240

Task 3C5:

Guess what patterns would you see if the numbers were written in a 4 column grid. Where would the square numbers be?

Task 3C6:

Can you make grids with other number of columns? Can you predict where the square numbers would be with such grids?

Task 4:

i) Without evaluating the square root can you figure out if the following are perfect squares? Also explain how you came to your decision.

a) 122 b) 356 c) 340 d) 12356 e) 425

ii) Give an example of a number that is a NOT perfect square and

a) whose units digit is 0, leaves a remainder 4 when divided by 8

b) whose units digit is 6, leaves a remainder 4 when divided by 8 and leaves a remainder 1 when divided by 3.

References:

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